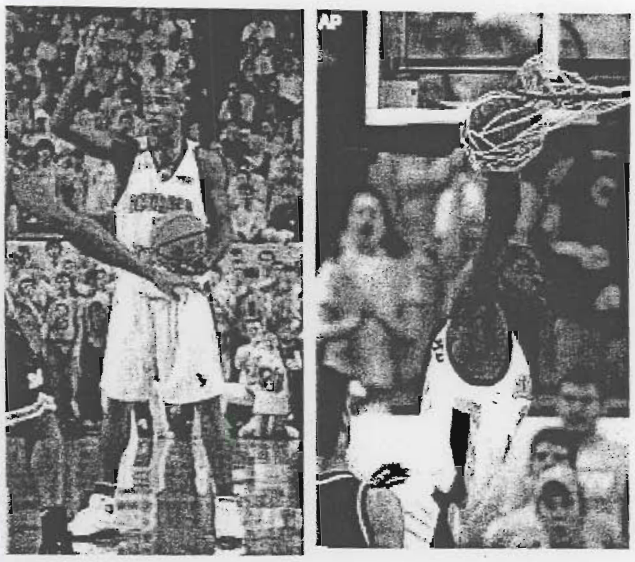
COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Spring 2005

GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENG.
GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GA 30332-0405

De Oliviera, Rosen (Chair), and Sitaraman

- All questions in this exam have a common theme: ***Basketball***

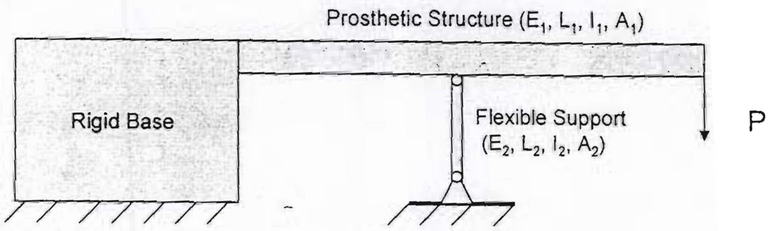


- Answer **all** questions.
- Manage your time carefully.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1
Finite Element Modeling

A design of prosthetic structure to aid players in Special Olympics basketball is shown in the figure below:



For analysis purpose, the prosthetic structure can be assumed to be fixed to a rigid base. The prosthetic structure has a modulus of elasticity E_1, length L_1, area moment of inertia I_1, and cross-section area of A_1. A flexible spring-like support is attached at the center of the prosthetic structure at a distance of $0.5L_1$ from the rigid base. The flexible support is pinned at both ends and has a modulus of elasticity E_2, length L_2, area moment of inertia I_2, and cross-section area of A_2. Assume that a force P is exerted at the tip of the prosthetic structure as shown in the figure.

- You are asked by the design team to use the finite-element method to determine the **vertical deflection and angle** of the tip of the prosthetic structure where *P* is applied. The design team is NOT INTERESTED in the stress/strain distribution in the prosthetic structure or the flexible support.
- Stiffness matrices for selected finite elements are given below. Choose appropriate element(s).
- You are asked to use minimum number of elements.
- Show all of your steps and solve for vertical deflection and angle at the tip of the prosthetic structure in terms of E_1, E_2, L_1, L_2, etc.
- State all your assumptions clearly.

Element A - Stiffness Matrix

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \qquad l=\frac{x_2-x_1}{L} \qquad m=\frac{y_2-y_1}{L}$$

where *E, A,* and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines of the element with respect

Element B - Stiffness Matrix

$$[K]=\frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where *E, I,* and *h* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Question 2
Geometric Modeling

When Is'mail Muhammad or other GT basketball players slam dunk the ball, they exert tremendous force on the rim of the basketball goal. You are to develop Bezier curve models of the rim that enable a designer to model undeflected and deflected shapes of the rim.

The equation for a Bezier curve is included at right.

$$b(u) = \sum_{i=0}^{n} B_{i,n}(u)\vec{P}$$

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

Answer the following questions:

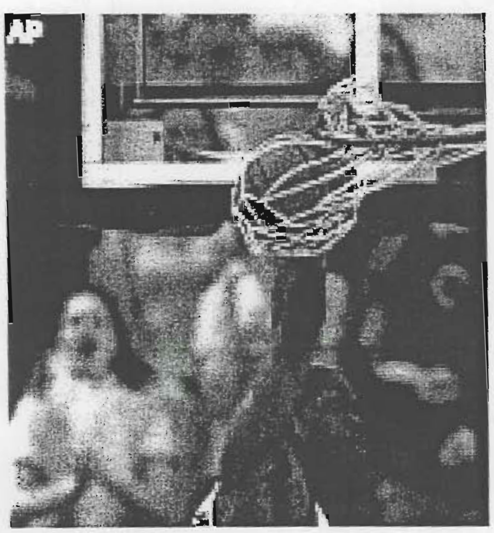
a. Sketch a circle representing the basketball goal rim. Show how to model approximately the (circular) rim using cubic Bezier curves. Sketch the curves and their control vertices.

b. It is necessary to ensure C0 and C1 continuity between Bezier curve segments. Describe how you will ensure this continuity. Upon a slam dunk, the rim will deflect out of the plane (you need a 3D curve to model 3D deflections).

c. Illustrate the usage of your continuity maintenance approach by sketching a bent rim (exaggerate the deflection) and showing how your control vertices would be displaced. Describe how your model ensures continuity.

d. Which type of curve (Hermite, Bezier, b-spline, etc.; 2^{nd}, 3^{rd}, 4^{th}, etc. order) would you use to enable a more accurate model of the rim? Justify your choice.

e. As you know, Bezier curves can only approximate the shape of a circular arc. Derive the equation of a cubic Bezier curve that best approximates a quarter circle. Sketch the problem. Identify the key unknowns of the problem. Explain your approach to solving the problem, then proceed to solve it.



Question 3

Numerical Methods

The Georgia Tech basketball team needs desperately a win in their last game in order to have a chance to go to the NCAA finals. The opponents are their arch-rivals University of Georgia who worryingly have the world's best young 3-pointer shooter. To neutralize him, the Alexander Memorial Coliseum is sealed, and the density of the air inside raised to 3 times its normal value. It is hoped that air resistance will hopefully become a major factor, thus unhinging the rival's expert shooter. A schematic of a 3-point throw is presented in Figure 1. The precision of throw can be calculated by solving the equations of motion of a basketball of mass m influenced by gravity g and air resistance:

$$\begin{cases} a_x = \dfrac{dV_x}{dt} = -\dfrac{1}{m}F_D\cos\theta \\ a_y = \dfrac{dV_y}{dt} = -\dfrac{1}{m}(F_D\sin\theta + mg) \quad \text{where } V = \sqrt{V_x^2 + V_y^2} \text{ and } \theta = \tan^{-1}\left(\dfrac{V_y}{V_x}\right) \\ F_D = \dfrac{1}{2}\rho A C_D V^2 \quad \text{(Drag force)} \end{cases}$$

The drag force F_D acting on the ball depends on air density ρ, velocity v, cross section area A and drag coefficient C_D.

$V_0 = 10.0$ m/s (throw velocity) $\theta_0 = 45^0$ (throw angle) $H_0 = 2.5$m (throw height)
$m = 0.6$kg $A = 4.52\text{x}10^{-2}$ m^2 $C_D = 0.5$
$\rho = 3.675$ kg/m^3 $g = 9.8$m/s^2

(a) The analytical formula for determination of height as function of horizontal distance x is given by:

$$y(x) = \frac{H_{max}\,x(L-x)}{x_a^2 + (L-2x_a)x}$$

where:

$$H_{max} = \frac{V_0^2\sin^2\theta_0}{g(2+kV_0^2\sin\theta_0)} \qquad V_a = \frac{V_0\cos\theta_0}{\sqrt{1+kV_0^2(\sin\theta_0+\cos^2\theta_0\ln\tan(\frac{\theta_0}{2}+\frac{\pi}{4}))}}$$

$$T = 2\sqrt{\frac{2H_{max}}{g}}, \qquad L = V_aT, \qquad x_a = \sqrt{LH_{max}\cot\theta_0}, \qquad k = \frac{\rho C_D A}{2mg}$$

H_{max} = max height reached by the ball
V_a = horizontal velocity of ball at H_{max}
x_a = x coordinate of ball at H_{max}
L = total length of shot if the basket was not there
T = time for the shot of length L

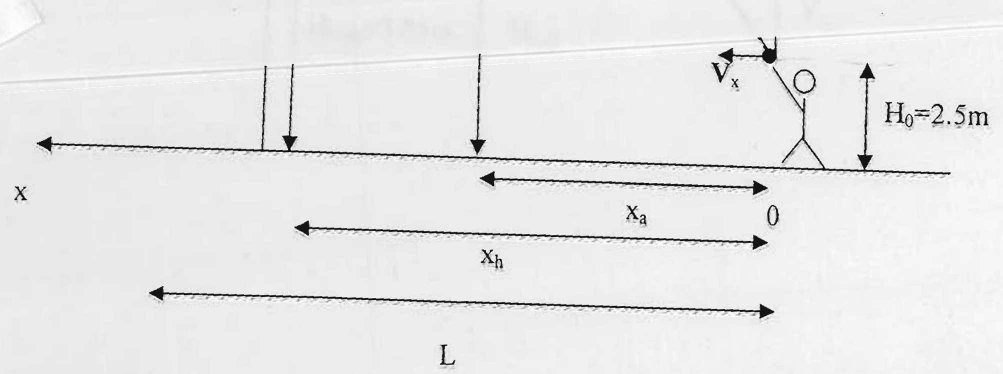


Figure 1 Schematic of a basketball throw, indicating the parameters used in calculating x_h, the horizontal distance between the player and the hoop.