COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – SPRING 2011

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- All questions in this exam have a common theme: ***Winter Weather and Snow Storm***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

1) Geometric Modeling

Suppose that you are a design engineer in a heavy equipment manufacturing company that builds snow plows. Your current task is to design the blade for a truck.

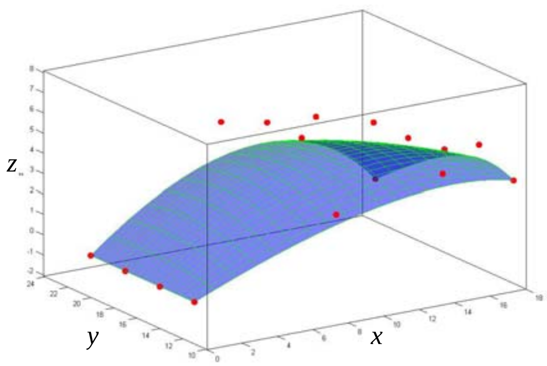
The first step in this design is to divide the blade into several segments and to model the segments using cubic Bézier surface patches.

a) The general form of the Bézier surface patch is $\vec{p}\left(u,w\right)=\sum_{i=0}^{m}\sum_{j=0}^{n}B_{i,m}\left(u\right)B_{j,n}\left(w\right)\vec{p}_{ij}$ where $B_{i,n}\left(u\right)=\begin{pmatrix}n\\i\end{pmatrix}u^{i}\left(1-u\right)^{n-i}$. Derive the equation of the cubic Bézier surface patch in a matrix form.

b) The first Bezier patch can be modeled using the following control points:

patch 1:

(0 20 0);	(8 21 5);	(14 22 4);	(18 23 0);
(0 17 0);	(8 17 6);	(14 17 5);	(18 17 3);
(0 14 0);	(8 14 6);	(14 14 5);	(18 14 4);
(0 11 0);	(8 11 3);	(14 11 4);	(18 11 3);



We need to check the normal directions on the surface to ensure that the shoveled snow can move to the right direction. What is the unit normal vector at the point (18 23 0)?

c) For the design of the second patch, you decided to add other control points as:

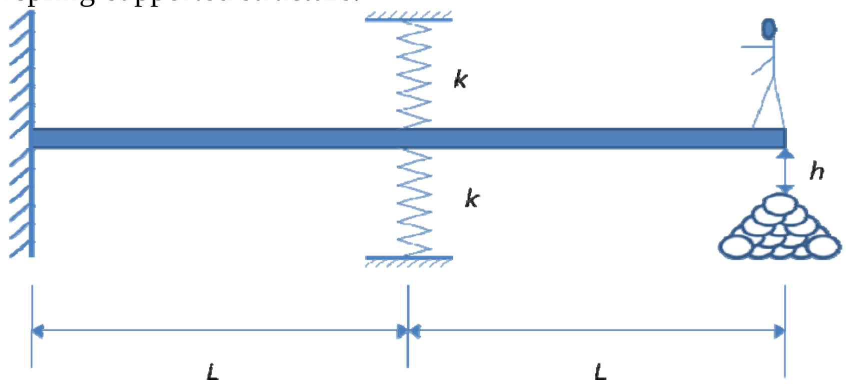
patch 2:

(0 11 0);	$\mathbf{P}_1$;	$\mathbf{P}_2$;	(18 11 3);
$\mathbf{P}_3$;	$\mathbf{P}_4$;	$\mathbf{P}_5$;	$\mathbf{P}_6$;
(0 4 0);	(8 4 1);	(14 4 2);	(18 4 2);
(0 0 0);	(8 0 0);	(14 0 0);	(18 0 0);

To ensure the G^1 continuity between the two patches, what are the coordinate values of $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$, $\mathbf{P}_5$, and $\mathbf{P}_6$? How do you decide?

2) Finite-Element Analysis

Figure shows a kid playing on a spring-supported structure.



The structure has a length of *2L* and a square cross-section with a side of *a*. The structure is made of a material with a modulus of elasticity *E*. The structure is rigidly clamped on one end and is free at the other end. The structure is also supported in the middle by two springs with a stiffness of *k* for each spring. As illustrated in the figure, a kid of mass *M* is standing at the tip of the structure, and a vertical heap of hardened snow is below the tip of the structure at a distance of *h*. You are asked to analyze the structure using finite-element formulation.

1. State all of your assumptions clearly.
2. Show all of your calculations.
3. Show the boundary conditions and loading conditions.
4. Write down element stiffness matrix and assembly stiffness matrix.
5. Determine the vertical deflection of the structure at the tip, assuming that the tip does not touch the hardened snow. Show all appropriate steps.
6. Now assume that an older kid with a mass of *2M* is standing at the tip of the structure and that the structure touches the "rigid" snow. Determine the deflection of the structure where the springs are attached.

Element A - Stiffness Matrix

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where *E, A*, and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines.

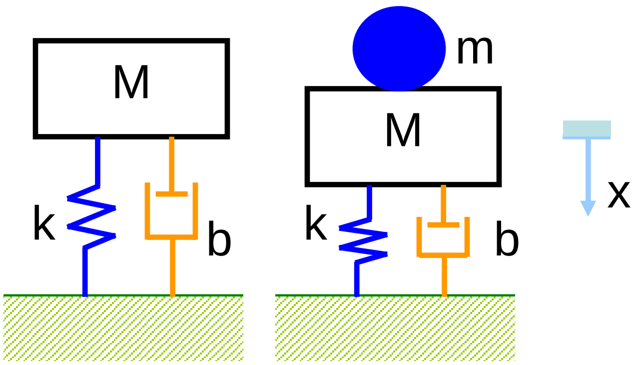
Element B - Stiffness Matrix

$$[K]=\frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where *E, I*, and *h* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

3) Numerical Methods

Due to the heavy snowstorm, the neighbor's tree fell on your car. The car mass, M=800kg, is supported by a spring (k=40,000N/m) and a damper (b=4,000Ns/m) as shown in the figure below. Assuming that at t=0, a mass of m=200kg is gently placed on the top of mass M, causing the car to exhibit vibrations. The displacement x of the combined mass is measured from the equilibrium position before m is placed on M. The gravitational acceleration, g, is 10m/s^2.



1. Obtain the equation of the motion for the given problem.
2. Write down the equivalent system of two first-order equations.
3. Determine approximate values of the solution, x, at the point t=0.2. Use the Euler method with h=0.1 and the Runge-Kutta method (RK4) with h=0.2. Compare the results with the values of the exact solution:

$$x(t) = 0.05(1 - \frac{1}{3}e^{-2t}\sin 6t - e^{-2t}\cos 6t)m$$

4. If you reduce the step size of the RK4 method as h=0.1, what is the expected error compared to the case of the step size, h=0.2 without resolving the ODE?
5. When we have a case where the exact solution is not smooth, what is your expectation of the accuracy obtained from both methods? If you want to improve the accuracy of this non-smooth solution problem with a numerical procedure, what would be your potential strategy?

Note: The equations for RK4 method are:

$$x_{n+1} = x_n + (h/6)(k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4})$$

$$k_{n1} = f(t_n, x_n),\ k_{n2} = f(t_n + (h/2),\ x_n + (h/2)k_{n1}),\ k_{n3} = f(t_n + (h/2),\ x_n + (h/2)k_{n2})$$

$$k_{n4} = f(t_n + h,\ x_n + hk_{n3})$$

The Euler formula is $x_{n+1} = x_n + hf_n$