GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 2004

APPLIED MATH

EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

* Please sign your **name** on the back of this page —

1. Consider a vector field $\bar{u}$ in a bounded region D defined by a volume V with a smooth closed boundary S.

(a) Write the divergence theorem.

(b) Consider a continuous scalar function ϕ in the domain D such that $\bar{u} = \nabla\phi$. Show that, if ϕ is a solution of Laplace's equation $\nabla^2\phi = 0$, the integral of the normal derivative of ϕ over S is zero. (Hint: Start from the divergence theorem).

(c) Let $\phi(x, y, z) = 2x^2 + 3y^2 + z^2$. Compute the directional derivative of ϕ in the direction of the vector $\vec{a} = \vec{i} - 2\vec{k}$, evaluated at the point P(2,1,3).

2. The acceleration of an object is a function of time t and its velocity V, as shown in the following equation:

$$\frac{dV}{dt} = t + \sqrt{V}.$$

The initial velocity (at $t = 0$) of the object is 100 m/s. It is of interest to find its velocity at $t = 4$ s.

(a) Solve the problem using Euler method with a step size of 4 s.

(b) Solve the same problem using the Trapezoidal rule with the same step size.

(c) Which method yields the more accurate result and why?

3. The upper end of a spring is attached to the ceiling. A mass, *m*, equal to 0.5 kg is suspended from the lower end of the spring. The spring constant, *k*, is 10 N/m. In parallel with the spring is a damper with a damping coefficient *c*, which produces a force equal to 2 times the velocity of the mass. The mass is initially at rest in its equilibrium position. At time $t = 0$ an external force $F(t) = 5\cos(2t)$ is applied to the mass. The equation describing the motion of the system is given by the linear differential equation:

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F(t).$$

(a) Find the general expression for $x(t)$.

(b) What restriction(s) must be placed on the parameters *m*, *c*, and *k* so that the system's transient response has no sinusoidal component?

(c) If the above equation were changed so it had the form below, is $t = 0$ a regular or an irregular singular point and why?

$$mt\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=0$$

4. An elastic bubble in the (x, y, z)-space is originally in the shape described by $x^2 + y^2 + z^2 = 1$. It is blown up in such a way that every point (x, y, z) on the bubble goes to (x_1, y_1, z_1) according to the transformation,

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Find the point(s) (x, y, z) *after* the transformation that are located at a minimum or maximum distance from the origin $(0,0,0)$ compared to all other points on the bubble. Furthermore, what are the new location(s) of the point(s) after the transformation? In your work, clearly explain the reason for every step of your procedure. The mathematics without a clear explanation of <u>why</u> each step was done will not count.

(Hint: $\lambda^3 - 8\lambda^2 + 19\lambda - 12 = (\lambda - 1)(\lambda - 3)(\lambda - 4)$.)