GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1996

APPLIED MATHEMATICS

EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

-- **Please sign your name on the back of this page** --

ANSWER ALL FIVE QUESTIONS---

1. (a) Derive the 2-D Newton Raphson algorithm for finding roots of a pair of nonlinear equations. Explain each step and define all terms.

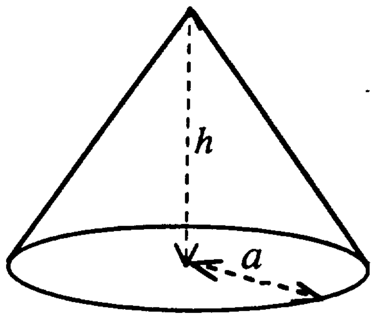
(b) Given: $f_1(x,y) = 4 - x^2 - y^2$

$f_2(x,y) = 1 - xy$

Using the 2-D Newton-Raphson method:

(i) Find (x_1,y_1), given $(x_o,y_o) = (2,0)$

(ii) Determine analytically the root nearest (2,0).



2. a. Use the divergence theorem to find the volume of a cone whose base has a radius a and whoe height is h. [Hint: Use the position vector $\boldsymbol{R} = (x,y,z)$ with the origin at the vertex of the the cone.]

b. Prove that for any vector **A**

$$\nabla \cdot (\nabla \times \mathbf{A}) = \mathbf{0}$$

3. Use the method of variation of parameters to find the general solution of the equation

$$y'' - \frac{4}{x} y' + \frac{4}{x^2} y = x^2 + 1$$

4. Find the solution of the equation

$$u_{xx} + 4u_{xy} + 4u_{yy} = 0$$

satisfying the boundary conditions

$$u(0,y) = f(y) \quad , u_y(x,0) = g(x)$$

(Hint: use the transformation

$$\xi = x \quad , \eta = -2x + y$$

to put the pde in canonical form)

5. Consider the exponential function of matrix **A** as defined below

$$\exp(\mathbf{A}) \equiv \sum_{k=0}^{\infty} \frac{\mathbf{A}^k}{k!} = \mathbf{I} + \mathbf{A} + \frac{\mathbf{A}^2}{2!} + \cdots + \frac{\mathbf{A}^m}{m!} + \cdots$$

where **I** is the identity matrix, **A** is an $n \times n$ matrix, and according to convention, $\mathbf{A}^2 = \mathbf{AA}$, $\mathbf{A}^0 = \mathbf{I}$, $0! = 1$.

Obviously, it is impossible to calculate each element in $\exp(\mathbf{A})$ directly from the definition because it is an infinite series.

(a) Please describe an alternative approach to obtain $\exp(\mathbf{A})$ through only *finite* number of calculations (Hint: consider the eigenvalues and eigenvectors of **A**).

(b) Please use the method you just described above to find $\exp(\mathbf{A})$ when

$$\mathbf{A} = \begin{bmatrix} 3 & 5 \\ -5 & 3 \end{bmatrix}.$$