GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2003

Applied Math
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

1. Let the matrix $\mathbf{A} = \mathbf{a}\mathbf{b}^T$, where **a** and **b** are (non-zero) $n \times 1$ column vectors. Find ALL of the eigenvalues and eigenvectors of **A**.

2. (a) Employ three-point Gauss quadrature to evaluate the following integral.

$$\int_1^4 xe^x dx$$

(b) If the integrand in the above integral were a fifth-order polynomial, would you expect to get an exact result when using three-point Gauss quadrature? Why or why not?

(c) Consider the following integral.

$$\int_0^1 \ln x \, dx$$

Is this integral well defined, i.e., does this integral have a finite value?

Which of the following numerical integration schemes can be used to evaluate this integral? Among these, which scheme would you choose? Why?

(i) Trapezoidal rule
(ii) Midpoint rule
(iii) Simpson's 1/3 rule
(iv) Three-point Gauss quadrature

Hint: Three-point Gauss quadrature is the following scheme.

$$\int_{-1}^{1} f(x)dx = c_0 f(x_0) + c_1 f(x_1) + c_2 f(x_2)$$

$c_0 = 0.5556$	$x_0 = -0.7746$
$c_1 = 0.8889$	$x_1 = 0.0$
$c_2 = 0.5556$	$x_2 = 0.7746$

3. (a) Solve the following initial-value problem.

$$u_t = ku_{xx}, \qquad 0 < x < l, \quad t > 0$$
$$u(0,t) = 0, \qquad t \geq 0$$
$$u(l,t) = 0, \qquad t \geq 0$$
$$u(x,0) = U_0 \sin\left(\frac{\pi x}{l}\right), \quad 0 < x < l$$

All quantities are real-valued and k is a positive constant.
The subscript refers to a partial derivative.

(b) How would you approach the problem if the boundary condition at $x = l$ was $u(l,t) = U_0$ and the initial condition was $u(x,0) = f(x), \quad 0 < x < l$, where $f(x)$ is a known function.

4. A mass is suspended by a spring and then driven from rest by a force. The appropriate equation of motion and its initial conditions are:

$$\frac{d^2y}{dt^2} + 64y = 16\cos(8t), \quad y(0) = \frac{dy}{dt}(0) = 0,$$

where y is the displacement of the mass and t is time. Determine $y(t)$ and sketch its graph.