Problem 1:

Let the vector $\bar{G}(x,y) = \left(xe^{x^2+y^2} + 2xy\right)\hat{i} + \left(ye^{x^2+y^2} + x^2\right)\hat{j}$

a. Show that

   $\bar{G}(x,y) = \nabla f$ for some *f* and find such an *f*.

b. If $\bar{G}$ is a force moving an object along the edges of a square with vertices (0,0), (0,1), (1,1) and (1,0) what is the work done by the force? Explain why the results make sense.

Problem 2:

Consider the following real-valued function:

$$f(x) = \exp(-\pi x^2)$$

Answer the following questions:

1. Find Fourier Transform of $f(x)$, i.e., $F(\omega) = \Im\{f(x)\}$? (<u>hints</u>: differentiation of $F(\omega)$ may be useful, and the value of Gauss integral is known to be $\int_{-\infty}^{\infty} \exp(-x^2)dx = \sqrt{\pi}$)
2. Comment on the condition required for the validity of the $F(\omega)$ differentiation approach utilized in answering the previous question?

Problem 3:

Determine a solution to the differential equation

$$2x^2y'' - xy' + (1+x)y = 0$$

near the regular singular point $x = 0$ using the method of Frobenius, i.e., construct a solution of the form

$$y = x^r \sum_{n=0}^{\infty} a_n x^n .$$

Problem 4:

Consider the following system of nonlinear equations:

$$f(x,y)=2x+2y-e^{xy}=0$$

$$g(x,y)=x^3+y-xy^3=1$$

a) In your opinion, is it possible to solve for x and y in closed form. If not, suggest a numerical algorithm for solving this system numerically.

b) Show how you would implement your suggested algorithm in (a) and carry out the first step of it starting with an initial guess of (0,0).

c) Can your algorithm in (b) find all the solutions?

d) Are there initial guesses for which the algorithm might diverge? If possible, give a precise condition to characterize all such 'bad' initial guesses.