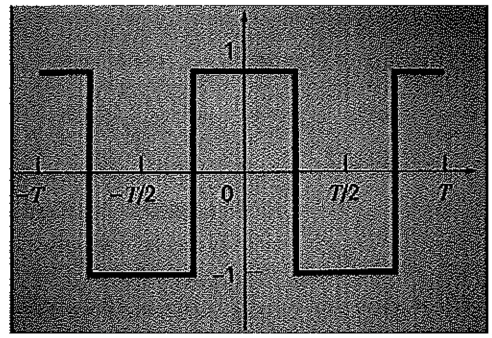
INSTRUCTIONS:

Please choose three of the four problems to work on. If you decide to try all four problems, you must clearly indicate which three you want graded – failure to do so will cause the three lowest scores to be used.

Problem 1

Use the continuous Fourier series to approximate the rectangular wave function

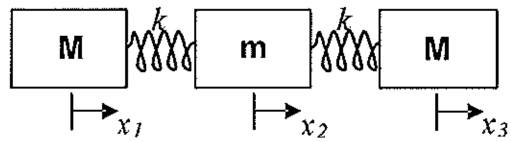
$$f(t)=\begin{cases}-1 & -T/2<t<-T/4\\ 1 & -T/4<t<T/4\\ -1 & T/4<t<T/2\end{cases}$$



Sketch the approximate wave form by keeping the Fourier series up to the first two terms and the first three terms, respectively.

Problem 2

Consider three masses (with the smaller mass m in the middle) constrained on the x-axis and joined by identical springs as shown in the figure to the right. The springs are assumed to be linear and follow Hooke's Law.



If we describe the motion with different coordinates for each mass, Newton's second law yields the following set of equations.

$$\ddot{x}_1 = -\frac{k}{M}(x_1 - x_2)$$

$$\ddot{x}_2 = -\frac{k}{m}(x_2 - x_1) - \frac{k}{m}(x_2 - x_3)$$

$$\ddot{x}_3 = -\frac{k}{M}(x_3 - x_2)$$

For the vibrating system of masses, find the common frequencies, ω, such that all masses vibrate at this same frequency. Comment briefly on the physical significance of the results.

Problem 3

Given the equation:

$$(3xy + y^2) + (x^2 + xy)\frac{dy}{dx} = 0$$

a) Show that the equation is not an exact equation (an equation in the form d/dx (F(x,y)) = 0)
b) Find an integrating factor (a term $\mu(x)$ to multiply by the whole equation) that makes this problem into an exact equation. Then solve the equation.

Problem 4

The parametric forms of two lines in space are,

$$\mathbf{r} = \mathbf{a}s + \mathbf{b}$$
$$\mathbf{r} = \mathbf{c}t + \mathbf{d}$$

where **a** and **c** are unit vectors, s and t are variable parameters.

a) If the two lines intersect at a single point find a necessary condition in terms of the vectors **a**, **b**, **c**, and **d**.
b) Explain the physical meaning of the condition in a) (a figure could be useful).
c) Determine the intersection point in a)
d) If the lines neither intersect nor are parallel (i.e. skew), find the shortest distance between them. (Hint: it's along the common normal.)
e) If the two lines in d) are parallel but do not intersect, find the shortest distance.