ACOUSTICS QUALIFYING EXAM

SPRING 2013

Work any 3 out of the following 4 problems.
Each problem will be graded on a 10-pt scale.

Problem 1

A narrow band of noise arrives at a receiver from a single, stationary source but traveling along two different paths. Using expressions for the total, combined acoustic pressure at the receiver (p_t), and the phase difference ($\Delta\beta$) between the two signals as a function of the path length difference:

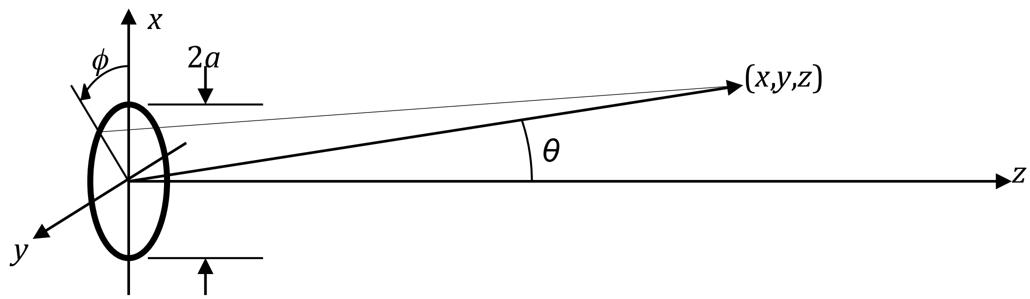
a. Write an expression for the sound pressure at the receiver (p_t) when the path length difference is 0.5 wavelengths.

b. Now assume that you measure 70 and 65 dB (*re* 20 μPa) at the receiver from paths 1 & 2, respectively, when the path length difference is 0.5 wavelengths. What is the total sound pressure level of the combination of the two waves?

c. Now assume that the path difference is at least 1 wavelength or greater and the bandwidth is wide enough for the two signals to each contain all phases. Show that the limiting form of the equation for total pressure (p_t) for this case corresponds to an expression for the addition of two separate sound signals of differing frequency content.

Problem 2

One of the noise sources associated with helicopters is the unsteady loading of the rotating blades when observed in a stationary frame of reference. Given that blade-loading commonly increases toward the blade tip, a potentially-useful first-cut analysis of radiated sound from a helicopter can be obtained by considering radiation from a ring source having the same radius, a, as the rotating-blade disk, and having an overall source strength of Q_o at radian frequency ω.



a) (2 pts) Determine an exact formula for the acoustic pressure at $\vec{r} = (x,y,z)$ in terms of the fluid density ρ, speed of sound c, radian frequency ω, Q_o, and x, y, z. You may use $k = \omega/c$ in your answer too. [Note: your formula should include an integral which you don't need to evaluate]

b) (2 pts) For a point on the acoustic axis (the z-axis) of the ring source show that the general formula from (a) reduces to:

$$\underline{p}(0,0,z,t) = \frac{j\rho\omega Q_o}{4\pi\sqrt{a^2+z^2}} \exp\left[j\omega\left(t - \sqrt{a^2+z^2}/c\right)\right]$$

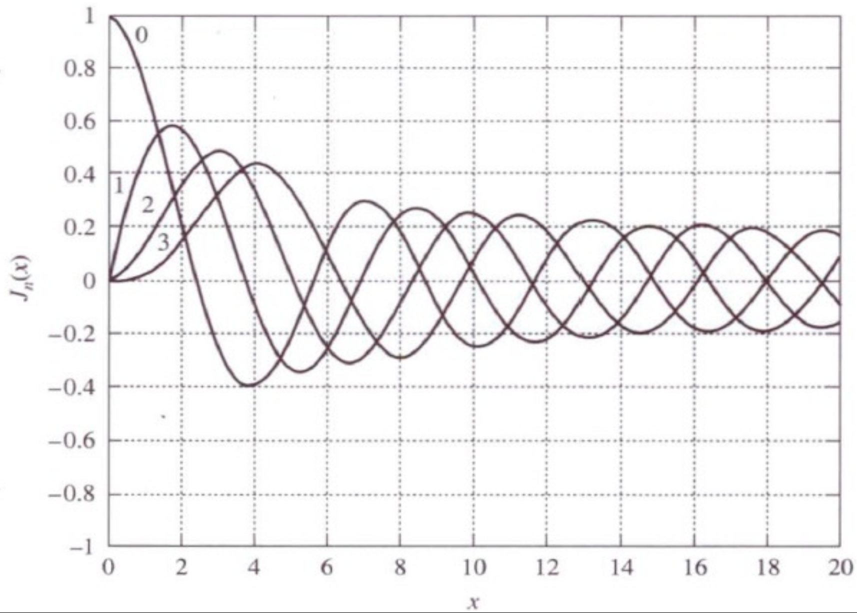
c) (1 pts) Does the ring source produce any near field nulls on the acoustic axis?

d) (3 pts) Use the far-field approximation for $r = |\vec{r}| >> a$ with $x/r = \sin\theta$, to show that:

$$\underline{p}(x,0,z,t) = \underline{p}(r,\theta,t) \cong \frac{j\rho\omega Q_o}{4\pi r} J_o(ka\sin\theta)\exp\left[j\omega(t - r/c)\right]$$

where $J_o(\psi) = \frac{1}{2\pi}\int_{\phi=0}^{\phi=2\pi} e^{j\psi\cos\phi}d\phi$ is the Bessel function of the Firs kind of order zero.

e) (2 pts) For a frequency of 50 Hz, blade radius of a = 6 m, and c = 343 m/s, compute ka and sketch the polar plot beam pattern of the ring source. Be sure to note any important angles and specify (in dB) the amplitude difference between the main lobe and any side lobes. [The Bessel function J_o is plotted and its zeroes are given in the Supplement below.]



Graphs: Bessel Functions of the First Kind of Orders 0, 1, 2, and 3

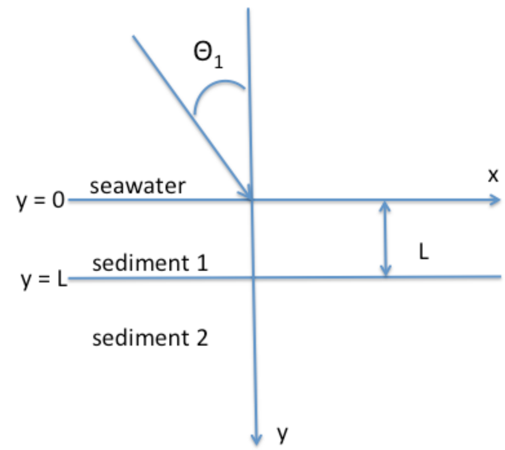
Zeros: Bessel Functions of the First Kind, $J_m(j_{mn}) = 0$

	j_{mn}				
m \ n	0	1	2	3	4
0	—	2.40	5.52	8.65	11.79
1	0	3.83	7.02	10.17	13.32

Problem 3

A harmonic plane wave propagating in seawater is incident on an interface consisting of two sediment layers. The sediments are assumed to be fluid-like. Properties for each layer are:

Fluid		ρc **(Pa-s/m)**	c **(m/s)**	ρ **(kg/m³)**
1	seawater	1.54×10^6	1500	1026
2	sediment 1	2.76×10^6	1540	1790
3	sediment 2	3.58×10^6	1730	2070



By applying continuity of the normal specific acoustic impedance at each interface, the reflection coefficient is found to be:

$$\tilde{R} = \frac{(\tilde{z}_2\tilde{z}_3 - \tilde{z}_1\tilde{z}_2)\cos(\tilde{k}_2 L) + j(\tilde{z}_2\tilde{z}_2 - \tilde{z}_1\tilde{z}_3)\sin(\tilde{k}_2 L)}{(\tilde{z}_2\tilde{z}_3 + \tilde{z}_1\tilde{z}_2)\cos(\tilde{k}_2 L) + j(\tilde{z}_2\tilde{z}_2 + \tilde{z}_1\tilde{z}_3)\sin(\tilde{k}_2 L)}$$

where the impedances, $\tilde{z}$, associated with each fluid layer and the wave number, $\tilde{k}_2$, are complex quantities that depend on the angle of incidence, Θ_1.

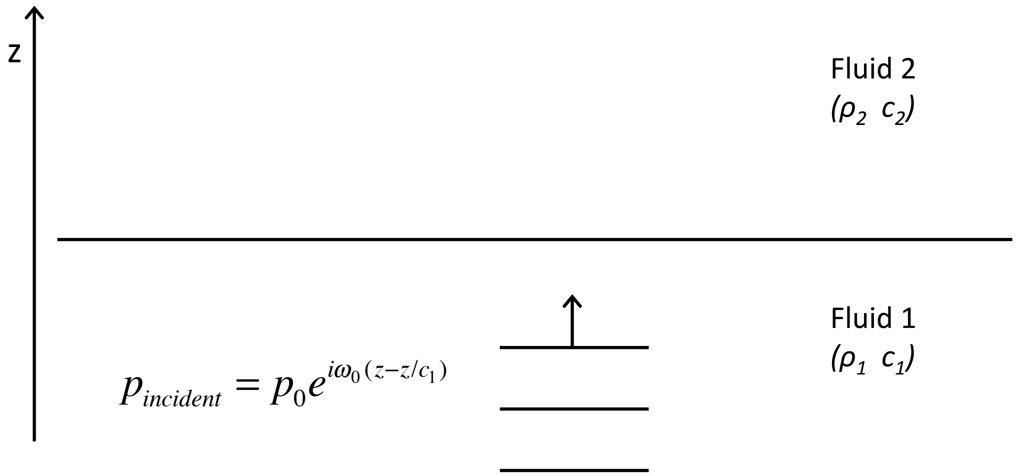
(a) What are $\tilde{z}_1, \tilde{z}_2, and\ \tilde{z}_3$ in terms of Θ_1?

(b) What is $\tilde{k}_2$ in terms of Θ_1?

(c) When is $\tilde{z}_2$ pure imaginary?

(d) Determine the angle of incidence beyond which the transmitted wave in sediment 2 cannot emerge at any real angle. What is the acoustic pressure in sediment 2 for this case in terms of Θ_1?

Problem 4



An acoustic field has a non-oscillatory "radiation pressure" which is equal to the time average of the acoustic energy density, $\langle E \rangle_{time}$, in the field:

1. Show that acoustic energy density has units of pressure. For a 1 atmosphere (10^5 Pa) travelling plane wave in water (ρ = 1000 kg/m^3, c = 1500 m/s), what is the magnitude (in Pa) of the radiation pressure? (1 point)

2. A high frequency acoustic wave propagates in the positive *z-direction,* at normal incidence to the interface of two immiscible liquids as shown above. The incident wave has amplitude p_0. The wave originates in liquid 1 (ρ_1, c_1) and propagates at normal incidence into the overlying liquid (ρ_2, c_2). In terms of these parameters, (p_0, ρ_1, ρ_2, c_1, and c_2), derive an expression for the radiation force/unit area acting on the interface between the two liquids. (7points)

3. Can this force ever be in the negative *z* direction? If not, why not? If so, give an example. (2 points)