PhD Qualifying Examination in Acoustics

Closed-book

Answer all parts of at least three questions

Problem #1 (This problem has four parts)
a) Assume that the sound pressure level due to white noise (equal energy per unit frequency) in octave band B is L_1. Find an expression for the total level for N octave bands that include band B. Recall that the bandwidth of a given octave band is twice that of the next lowest band, that is $BW_B = 2\text{x}BW_{B-1}$, where B is the band number.

b) Repeat part a), except assume pink noise (equal energy per octave band).

c) An acoustic pressure signal is in the form of a periodic square wave, $p=+A$ for a time interval $T/2$, then $p=-A$ then for a time interval $T/2$, etc., where A is constant. If the period T is 0.001 seconds, and the amplitude A is 1 Pa, what is the sound pressure level of this signal, dB re 20 μPa?

d) An acoustic pressure signal is in the form of a periodic square wave, $p=+A$ for a time interval $T/2$, then $p=-A$ then for a time interval $T/2$, etc., where A is constant. If the period T is 0.001 seconds, and the amplitude A is 1 Pa, what would be the octave-band sound pressure level (dB re 20 μPa) for the octave band centered at 1000 Hz? Recall that the lower and upper band limits for octave bands are related by $\sqrt{2}$ to the center frequency of the band (i.e. $f_{lower} = f_c / \sqrt{2}$, $f_{upper} = f_c \sqrt{2}$).

Problem #2 (This problem has four parts)
A circular loudspeaker of diameter 12.5 cm is mounted in a rigid baffle and driven, in air, at a frequency of 6550 Hz. Assume the speed of sound is 343 m/s, and ρ*c*= 400 Rayl.

a) Find the Rayleigh distance

b) How much acoustic power (mW) is radiated when the sound pressure level on axis is 93 dB at a distance of 1 m?

b) Find the half-power beamwidth (in the farfield)

d) Find the location of the most distant null on the axis

Work these problems as far as you can; clearly state all assumptions or approximations, and if possible, validate your approximations. You will receive full credit if your final expression(s) merely lack numerical evaluation.

$$p(x,y,z,t) = \frac{jk\rho_o c_o u_o e^{j\omega t}}{2\pi} \int_S \frac{e^{-jkR}}{R} dS$$

$$p(r,\theta,t) = \frac{ja\rho_o c_o u_o e^{j\omega t}}{r} \frac{ka}{2} e^{j(\omega t - kr)}$$

$$p = P_0 \left[e^{j(\omega t - kr)} - e^{j(\omega t - kr_1)} \right]$$

$$D(\theta) = \frac{2J_1(ka\sin\theta)}{ka\sin\theta}$$

$$\frac{2J_1(ka\sin\theta)}{ka\sin\theta} = \frac{1}{\sqrt{2}}$$

$$2\theta = 2\sin^{-1}(1.616/ka)$$

$$W = Su_{rms}^2 \operatorname{Re}(Z_p)$$

$$R_1(2ka) = 1 - \frac{2J_1(2ka)}{2ka}$$

$$W = \frac{\pi a^2}{2\rho c} P_0^2 R_1(2ka)$$

$$R_0 = S/\lambda = ka^2/2$$

$$p = \frac{P_0 R_0}{r} e^{j(\omega t - k(r - R_0))}$$

$$\sin\left(\pi a/\lambda \left(\sqrt{(a/r)^2 + 1} - (a/r) \right) \right) = 0$$

A6 TABLE OF DIRECTIVITIES AND IMPEDANCE FUNCTIONS FOR A PISTON

x	*Directivity Functions* ($x = ka \sin\theta$)		*Impedance Functions* ($x = 2ka$)	
	Pressure	*Intensity*	*Resistance*	*Resistance*
	$\frac{2J_1(x)}{x}$	$\left(\frac{2J_1(x)}{x}\right)^2$	$R_1(x)$	$X_1(x)$
0.0	1.0000	1.0000	0.0000	0.0000
0.2	0.9950	0.9900	0.0050	0.0847
0.4	0.9802	0.9608	0.0198	0.1680
0.6	0.9557	0.9134	0.0443	0.2486
0.8	0.9221	0.8503	0.0779	0.3253
1.0	0.8801	0.7746	0.1199	0.3969
1.2	0.8305	0.6897	0.1695	0.4624
1.4	0.7743	0.5995	0.2257	0.5207
1.6	0.7124	0.5075	0.2876	0.5713
1.8	0.6461	0.4174	0.3539	0.6134
2.0	0.5767	0.3326	0.4233	0.6468
2.2	0.5054	0.2554	0.4946	0.6711
2.4	0.4335	0.1879	0.5665	0.6862
2.6	0.3622	0.1326	0.6378	0.6925
2.8	0.2927	0.0857	0.7073	0.6903
3.0	0.2260	0.0511	0.7740	0.6800
3.2	0.1633	0.0267	0.8367	0.6623
3.4	0.1054	0.0111	0.8946	0.6381
3.6	0.0530	0.0028	0.9470	0.6081
3.8	+0.0068	0.00005	0.9932	0.5733
4.0	−0.0330	0.0011	1.0330	0.5349
4.5	−0.1027	0.0104	1.1027	0.4293
5.0	−0.1310	0.0172	1.1310	0.3232
5.5	−0.1242	0.0154	1.1242	0.2299
6.0	−0.0922	0.0085	1.0922	0.1594
6.5	−0.0473	0.0022	1.0473	0.1159
7.0	−0.0013	0.00000	1.0013	0.0989
7.5	+0.0361	0.0013	0.9639	0.1036
8.0	0.0587	0.0034	0.9413	0.1219
8.5	0.0643	0.0041	0.9357	0.1457
9.0	0.0545	0.0030	0.9455	0.1663
9.5	0.0339	0.0011	0.9661	0.1782
10.0	+0.0087	0.00008	0.9913	0.1784
10.5	−0.0150	0.0002	1.0150	0.1668
11.0	−0.0321	0.0010	1.0321	0.1464
11.5	−0.0397	0.0016	1.0397	0.1216
12.0	−0.0372	0.0014	1.0372	0.0973
12.5	−0.0265	0.0007	1.0265	0.0779
13.0	−0.0108	0.0001	1.0108	0.0662
13.5	+0.0056	0.00003	0.9944	0.0631
14.0	0.0191	0.0004	0.9809	0.0676
14.5	0.0267	0.0007	0.9733	0.0770
15.0	0.0273	0.0007	0.9727	0.0880
15.5	0.0216	0.0005	0.9784	0.0973
16.0	0.0113	0.0001	0.9887	0.1021

Problem #3 (This problem has two parts)

1) In many applications of acoustics the ambient fluid is not motionless. The simplest extension of the motionless-ambient-fluid formulation of acoustics, involves ambient fluid motion at a constant velocity $\vec{U}$ where $\vec{U} = U_x\hat{x} + U_y\hat{y} + U_z\hat{z}$ is a constant vector in both time and space. Under this assumption, demonstrate that the linearized acoustic wave equation in air for the pressure fluctuation p can be expressed as

$$\left[\frac{\partial}{\partial t} + \vec{U}\cdot\vec{\nabla}\right]\left[\frac{\partial}{\partial t} + \vec{U}\cdot\vec{\nabla}\right]p - c^2\nabla^2 p = 0 \quad \text{Eq. (1),}$$

starting from

- the continuity equation: $\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{u}) = 0$
- Euler's ideal flow momentum equation: $\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u} + \frac{1}{\rho}\vec{\nabla}P = 0$
- the constitutive equation: $\rho = \rho_o + p/c^2$

where ρ is the total density, $\vec{u}$ the total velocity of the fluid and ρ_o is reference density in absence of acoustic wave.

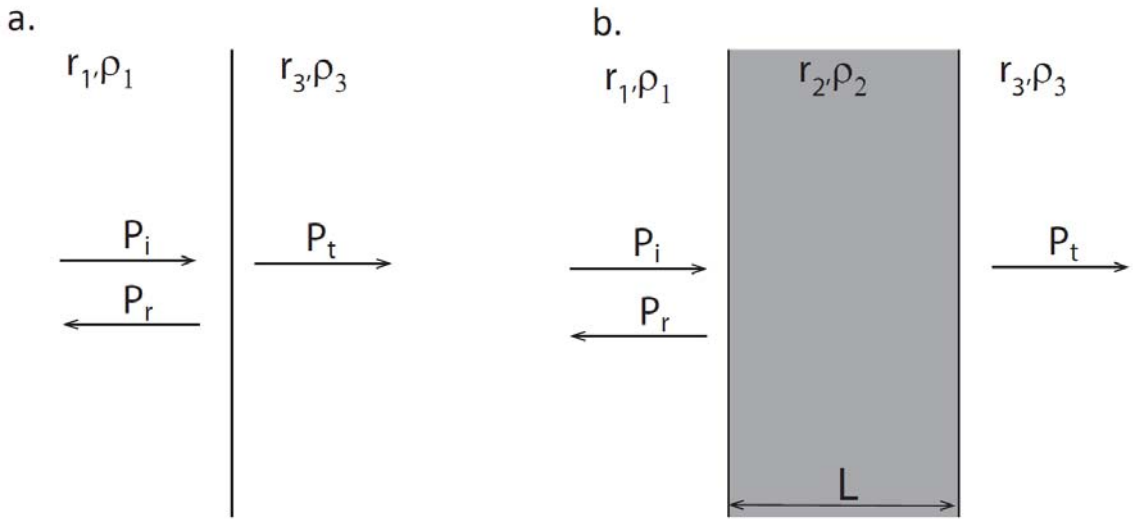
Assume that $\vec{U} = U_x\hat{x} + U_y\hat{y} + U_z\hat{z}$ is a constant vector in both time and space. Also Assume that the speed of sound, c, ambient static pressure P_o and reference density ρ_o are independent of time and constant everywhere.

[*Hint. The gradient operator,* $\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$ *in Cartesian coordinates, is merely a short hand notation for a vector derivative. It follows all the usual rules for differentiation except that it also has a vector character, so the order of dot-products needs to be respected, for example:*

$$\vec{U}\cdot\vec{\nabla} = U_x\frac{\partial}{\partial x} + U_y\frac{\partial}{\partial y} + U_z\frac{\partial}{\partial z} \neq \vec{\nabla}\cdot\vec{U} = \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z}.]$$

2) Obtain the dispersion relationship between the wavenumber k and the frequency ω for a harmonic plane wave solution of Eq. (1) and propagating along the x direction only. Assume $\vec{U} = U_x\hat{x}$.

Problem #4 (This problem has three parts)
A plane wave of frequency ω is normally incident on the boundary between two fluids of characteristic impedance r_1 and r_3 (Fig. a). The fluid densities of the fluids are ρ_1 and ρ_3. Assume that r_3=0.01 r_1.



(1) For the configuration in Fig. a, calculate the pressure transmission coefficient, T, and the intensity transmission coefficient T_I (2points)

(2) In order to increase the transmission of power at specific frequencies, a layer of thickness L is added between fluids 1 and 3 (Fig. b). This layer is made of a material of characteristic impedance r_2 and density ρ_2. Write a system of equations that need to be solved in order to find the pressure reflection coefficient and the pressure transmission coefficient. Clearly write the unknowns in the equations; the other terms should be expressed as a function of the given parameters. (3 points)

(3) If we solve this system of equations from part 2, we obtain the following expression for the pressure reflection coefficient:

$$R = \frac{(1 - r_1/r_3)\cos k_2 L + j(r_2/r_3 - r_1/r_2)\sin k_2 L}{(1 + r_1/r_3)\cos k_2 L + j(r_2/r_3 + r_1/r_2)\sin k_2 L}$$

If $L = \frac{\pi}{2k_2}$, calculate the intensity transmission coefficient T_I. Determine the value of r_2 that maximize the transmission of acoustic energy and the maximum value of T_I. (5 points)