GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1998

Acoustics
EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

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The Exam Committee will get a copy of this exam and will not be notified whose paper it is until it is graded.

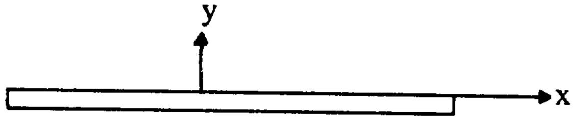
Acoustics Qualifying Exam, Spring, 1998

Work all 3 problems. Show all of your work. Clearly state all assumptions.

Propagation

The one-dimensional propagation of flexural waves, y(x,t), obeys the following "wave equation".

$$\frac{\partial^4 y}{\partial x^4} + \alpha^4 \frac{\partial^2 y}{\partial t^2} = 0$$



where α is a constant which depends on material properties, y represents the normal displacement, and x the axial distance. Note that this "wave equation" is actually not in the form of the standard wave equation. One of the consequences is that the speed of propagation of flexural waves depends on frequency (dispersion). Consequently, the wavenumber $k(\omega) = \omega / c(\omega)$ is a nonlinear function of frequency.

1) Assume a solution of the form $y(x,t) = A \exp(i\,\Phi)$, where the phase function of the wave is of the form $\Phi(x,t) = -\omega t + k(\omega)\,x$, i.e., where the wavenumber is a function of frequency. Find and sketch the dispersion relation (i.e., the $k(\omega)$ relation, and plotted as $k(\omega)$ vs. ω) for flexural propagation *and find* the frequency dependence of flexural wavespeed.

2) Consider now the superposition of two harmonic flexural waves traveling in the +x direction, each with the same amplitude A but with slightly different frequencies $\omega - d\omega/2$ and $\omega + d\omega/2$, i.e. with slightly different phase functions $\Phi = \Phi - d\Phi/2$ and $\Phi = \Phi + d\Phi/2$. Show that the resulting flexural wave is of the form

$$y(x,t) = 2A\, F[k(x - v_g t)] \exp[\, ik\,(x - v_p t)].$$

Find explicit expressions for v_g and v_p, in terms of k, ω, and $k(\omega)$ or $d\omega/dk$ and give a *physical* interpretation of the result.

Recall: $2 \cos x = \exp(ix) + \exp(-ix)$

A small sphere (radius $a \ll \lambda$) oscillates radially. This sphere is driven by an external source of energy, harmonically at an angular frequency ω, and *at constant source strength* (the amplitude, v_s, of the normal velocity at the surface of the sphere is constant)

(a) Consider the configuration shown in Figure (a), where the sphere radiates into a fluid of infinite extent. Derive the expression for the time-average acoustic power radiated by the sphere, *both* by considering the power flow in the far field *and* by considering the power flow at the surface of the monopole.

(b) The sphere is now placed near a rigid boundary, as shown in Figure (b). Show that the presence of the rigid boundary increases by a factor of two the acoustic power radiated by the sphere. Explain the physical origin of the increase in the radiated power and the source of this additional energy.

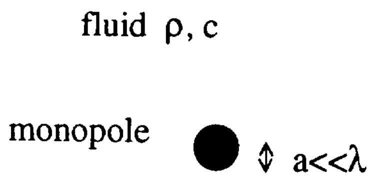


Figure (a)

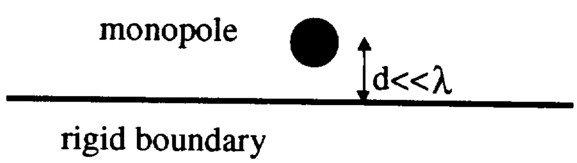


Figure (b)

The mechanical termination impedance (at $x=L$) of an open ended flanged circular tube of length $L=1$ m and radius $a=0.05$ m is

$$\frac{Z_{mL}}{\rho c S} = \frac{1}{2}(ka)^2 + j\frac{8}{3\pi}ka$$

where S is the cross-sectional area of the tube. Let $\rho c=415$ Rayl, and $c=343$ m/s.

A piston of mass m =0.015 kg at $x=0$ is driven at 150 Hz with a displacement amplitude of 0.005 m.

a) Determine the total mechanical impedance of the piston, including fluid loading (recall that mechanical impedance is defined as the ratio of force to velocity. Here, the impedance will include both the mechanical impedance of the piston plus the input impedance of the tube).

b) For the given displacement amplitude, determine the power radiated from the end of the tube.