Acoustics Qualifying Exam
Spring 2011
March 1,2011
Two Hours
Closed Book
Answer all three questions

1. A piston is mounted so as to radiate on one side of an infinite baffle into air. The radius of the piston is *a*, and it is driven at a frequency such that $\lambda = \pi a$ . Assume *a* = 0.1 m and the maximum displacement amplitude of the piston is 0.0002 m. The table below provides directivity and impedance functions for a piston.
   *a)* How much acoustic power is radiated?
   *b)* Recalling that $D = \frac{(ka)^2}{1 - J_1(2ka)/ka}$ for a piston source, what is the directivity index of the radiated beam?

| | Directivity Functions ($x = ka \sin\vartheta$) | | Impedance Functions ($x = 2ka$) | |
|---|---|---|---|---|
| $x$ | Pressure $\frac{2J_1(x)}{x}$ | Intensity $\left(\frac{2J_1(x)}{x}\right)^2$ | Resistance $R_1(x)$ | Resistance $X_1(x)$ |
| 1.0 | 0.8801 | 0.7746 | 0.1199 | 0.3969 |
| 2.0 | 0.5767 | 0.3326 | 0.4233 | 0.6468 |
| 3.0 | 0.2260 | 0.0511 | 0.7740 | 0.6800 |
| 4.0 | - 0.0330 | 0.0011 | 1.0330 | 0.5349 |
| 5.0 | - 0.1310 | 0.0172 | 1.1310 | 0.3232 |
| 6.0 | - 0.0922 | 0.0085 | 1.0922 | 0.1594 |

2. A spherically-expanding acoustic wave, $(\hat{A}/r)e^{i(kr-\omega t)}$, with radian frequency $\omega = ck$ propagates away from the origin of coordinates ($r$ = 0) in a fluid medium with density $\rho$ and speed of sound $c$. This wave is incident on a thin spherical membrane centered on the origin. A fraction of the incident wave is reflected as a spherically-contracting wave $\hat{R}(\hat{A}/r)e^{i(-kr-\omega t)}$, and a fraction is transmitted into an identical fluid medium outside the membrane as a spherically-expanding wave $\hat{\tau}(\hat{A}/r)e^{i(kr-\omega t)}$. The incident sound wave causes the membrane to vibrate so that its radius is $a + \hat{\eta}e^{-i\omega t}$ where $|\hat{\eta}| << a$.

**a)** (10 pts) The first two boundary conditions on the membrane are:

*i*) Radial velocities must match at $r = a$: $(v_r)_{r=a^-} = (v_r)_{r=a^+}$, and

*ii*) Fluid and membrane motion must match at $r = a$;

$(v_r)_{r=a^+} = (\partial/\partial t)(a + \hat{\eta}e^{-i\omega t})$.

Use these two boundary conditions to establish the following relationships:

$$(1-\hat{\tau})\left(1-\frac{1}{ika}\right)e^{ika} = \hat{R}\left(1+\frac{1}{ika}\right)e^{-ika}, \text{ and } -i\omega\hat{\eta} = \frac{\hat{A}\hat{\tau}}{\rho c}\left(1-\frac{1}{ika}\right)\frac{e^{ika}}{a}$$

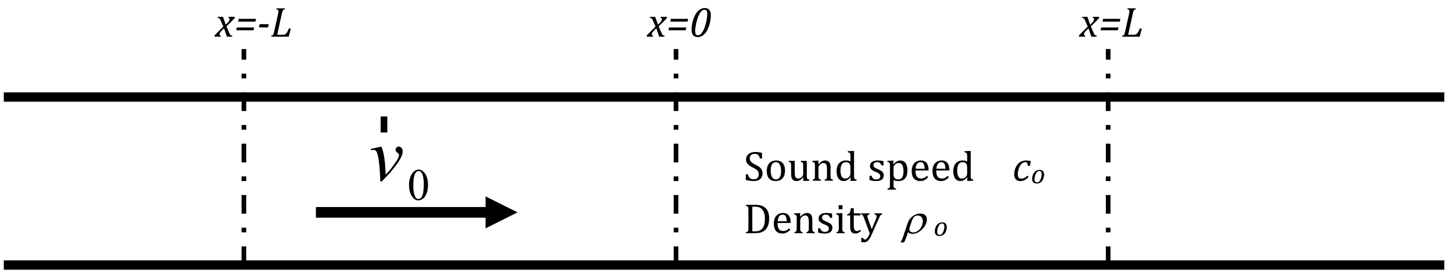
**b)** (20 pts) For this spherical geometry, the following linearized dynamic boundary condition on the membrane can be obtained :

$$iii)\ m_m \frac{\partial^2(a+\hat{\eta}e^{-i\omega t})}{\partial t^2} = (p)_{r=a^-} - (p)_{r=a^+} - 2\frac{T}{a^2}\hat{\eta}e^{-i\omega t},$$

where $m_m$ is the mass per unit area of the membrane, $p$ is the complex acoustic pressure, and $T$ is the tension in the membrane when it has radius $a$. The tension T is assumed to large enough to be considered constant. Use this boundary condition and the results of part a) to determine:

$$\hat{\tau} = \frac{1}{1 - \frac{i\omega m_m}{2\rho c}\left(1+\frac{1}{k^2a^2}\right)\left(1-\frac{1}{k^2a^2}\frac{2(T/m_m)}{c^2}\right)}$$

**c)** (5 pts) With $m_m$ and $T$ non-zero, under what condition is the membrane acoustically transparent? Describe with words what is happening physically in this case.




3. A rigid wall pipe is filled with a fluid with sound $c_o$ and density $\rho_o$. The fluid is flowing in the positive $x$-direction with a uniform velocity of $v_o$. The system can be considered to be one-dimensional.

   a. (1 point) What is the linearized Euler equation for this system?

   b. (1 point) What is the linearized continuity (mass conservation) equation?

   c. (1 point) The equation of state for this system is given by
   $$\frac{\partial p}{\partial t} + v_o \frac{\partial p}{\partial x} = c^2 \left( \frac{\partial \rho}{\partial t} + v_o \frac{\partial \rho}{\partial x} \right)$$
   Why does it depend on the flow velocity?

   d. (5 points) Derive the wave equation for this system

   e. (2 points) What is the phase velocity for this system