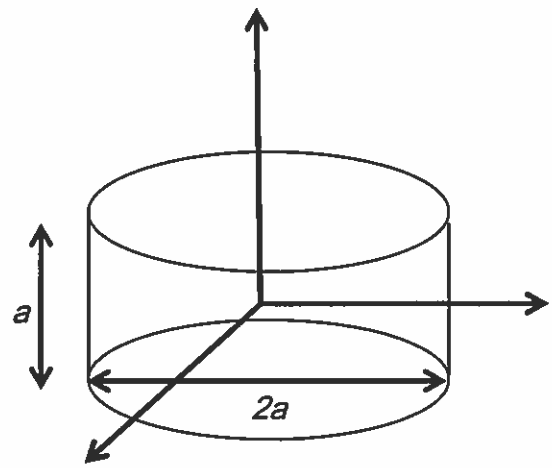
Problem #1



The center of a cylinder whose radius and height are both equal to a is located at the origin of a Cartesian coordinate system with its axis aligned with the z-axis as shown above. Define the angle θ as the elevation angle and ϕ as the azimuth angle so that $e.g.$ $x = R\sin\theta\cos\phi$. The top surface oscillates with a normal velocity given by $v_{n\,top} = v_0\cos(\omega t)$, the bottom surface oscillates with a normal velocity given by $v_{n\,bot} = v_0\cos(\omega t)$ and the side surface vibrates with a normal velocity given by $v_{n\,side} = -v_0\cos(\omega t)$. Assuming that $a \ll \lambda$,

a. Show that the radiated field has no monopole or dipole component. Mathematical and physical arguments are acceptable.

b. Show that the radiated beam pattern must be of the form

$$D(\theta,\phi) = (1+b)\cos^2\theta - b$$

c. Can the constant b be found? If so, how? If not, why not?

Problem #2

Two sources emit 63 Hz pure tones simultaneously and are out of phase by a constant 0.8 rad. The two sources are located x_1 and x_2 m from a receiver, inline with the receiver. A sensor measures sound pressure levels of $L_{P1} = 68$ and $L_{P2} = 72$ dB at the receiver. The medium is air at 20°C. Calculate:

1. The difference in distance between x_1 and x_2
2. The difference in time between the two waves
3. The total rms pressure ($p_{rms,tot}$) & total sound pressure level ($L_{P,tot}$) of the combination of the two waves @ the receiver.
4. Now assume that you adjust one of the two sources to emit a 150 Hz pure tone, but leave the other source unchanged. Does your calculation in part (c) above change? If so, what are the new total rms pressure ($p_{rms,tot}$) & total sound pressure level ($L_{P,tot}$) @ the receiver?

Problem #3

Consider a harmonic plane wave of fixed frequency ω propagating in the +*x* direction in a medium with mean flow velocity u_o also in the +*x* direction.

a) Derive the wave equation, given the linearized continuity equation,

$$\frac{\partial p}{\partial t} + u_o \frac{\partial p}{\partial x} + \rho_o c_o^2 \frac{\partial u}{\partial x} = 0$$

and the linearized momentum equation

$$\rho_o \left[\frac{\partial u}{\partial t} + u_o \frac{\partial u}{\partial x} \right] + \frac{\partial p}{\partial x} = 0$$

b) Show that the coordinate transformation

$$x' = x - u_o t$$

with

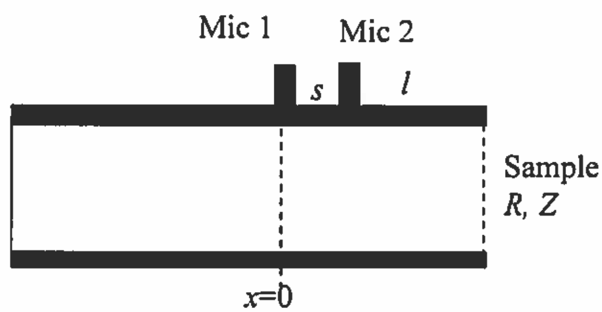
$$t' = t, \quad \frac{\partial p}{\partial x} = \frac{\partial p}{\partial x'}, \text{ and } \frac{\partial p}{\partial t} = \frac{\partial p}{\partial t'} - u_o \frac{\partial p}{\partial x'}$$

leads to a solution which indicates a Doppler shift in the frequency.

c) What is the relationship between mean flow velocity u_o and the change in frequency?

Problem #4

Consider the two-microphone impedance tube method for determination of the complex normal incidence reflection coefficient and acoustic impedance of a material. The method employs two microphones spaced a known distance s apart, and with l the known distance from the face of the material under test to the closest mic, as depicted in the figure.



1) Consider plane waves in the tube of frequency ω incident on the sample from the left. The measured transfer function between the two microphones may be represented as

$$H_{12}(\omega) = \frac{P_2(\omega)}{P_1(\omega)}$$

where P_1 and P_2 are the total pressures at mics 1 and 2. Derive an expression for the transfer function that is only a function of the incident pressure, the geometric parameters s and l of the tube, the reflection coefficient of the sample (which may be complex), and the wavenumber. Hint: consider that the total pressure at each mic is due to the superposition of incident and reflected waves.

2) What is the relationship between the reflection coefficient and the impedance at the termination of the tube?