PhD Qualifying Examination in Acoustics

Closed-book

Work All Questions

Problem #1 (This problem has three parts)

a) Consider the formal definition of the mean-square average pressure;

$$\langle p^2 \rangle_T = \frac{1}{T}\int_0^T p^2(t)\,dt$$

If $p(t)$ is a single-frequency harmonic wave of period T', but the averaging duration T is arbitray (i.e., $T = nT' + \varepsilon T'$, where $0 < \varepsilon < 1$ and n is an integer), derive an expression for the fractional error in $\langle p^2 \rangle_T$ as compared to the average obtained when T is identically an integer number of periods of the wave. That is, you're seeking a relationship to express

$$\delta = 1 - \frac{\langle p^2 \rangle_T}{\langle p^2 \rangle_{nT'}},$$

where δ represents the fractional error.

Recall that $\int \sin^2(ax)\,dx = \frac{1}{2}x - \frac{1}{2a}\cos(ax)\sin(ax) = \frac{1}{2}x - \frac{1}{4a}\sin(2ax)$

b) Find an expression that relates the minimum averaging duration T as a function of the period of the wave to ensure that the maximum fractional error is less than +/- 1%. You will have to make suitable approximations and simplifications, and clearly state them.

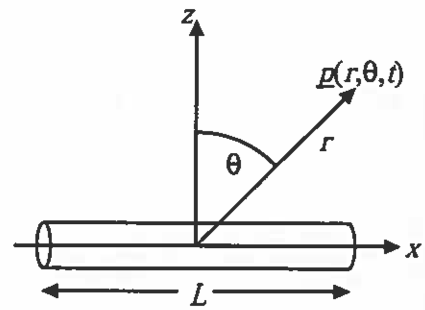
c) Comment on the implications of your results with respect to the relative accuracy of spectral power estimates obtained by use of the Fourier Transform with a fixed averaging duration T. That is, for a power spectrum obtained through a Fourier Transform (FFT, e.g.), with fixed averaging period T, what is the relative accuracy of the individual spectral components, and how does it depend on averaging time T? You might want to find an expression comparing the relative accuracy of two frequency components within a Fourier spectrum.

Problem #2 (This problem has four parts)

Consider a plane wave (in air) of frequency 1000 Hz and sound pressure level 100 dB SPL. A locally reacting acoustic tile is added such that the angle of incidence is θ_i=30 degrees. The normal acoustic impedance of the acoustic tile is z_n=300-j500 Pa.s/m. Use ρc= 415 Pa.s/m for air.

1. (3 points) Determine the value of the pressure reflection coefficient
2. (2 points) Determine the intensity reflection coefficient.
3. (4 points) What is the sound pressure level just at the surface of the tile?
4. (1 point) Does the surface absorb power? Justify your answer.

Problem #3 (this problem has 6 parts)



Linear transducer arrays are commonly used in directional microphones, biomedical ultrasound, and underwater acoustics. Here we will study how sound radiation from a continuous harmonic line source of length L and radius a can be altered when the radial surface velocity is not uniform along the length of the line source. Assume $ka \ll 1$ throughout this problem, and only consider radiated sound characteristics in the x-z plane.

NOTE: if you are running out of time, you may use the given result of part b) to answer the subsequent questions c)-f)

a) If the radial surface velocity on the surface of the line source is given by

$$\underline{u}_{surface}(x) = U_o e^{j\omega t}\underline{\Gamma}(x),$$

show that the radiated sound in the Fraunhofer far-field is:

$$\underline{p}(r,\theta,t) = \frac{j}{2r}\rho_o c U_o ka \exp\{j(\omega t - kr)\} \int_{-L/2}^{+L/2} \underline{\Gamma}(x)\exp\{jkx\sin\theta\}dx$$

Recall that the radiated pressure p at a distance r' by a monopole of acoustic strength $\underline{Q}$ is: $\underline{p} = j\dfrac{\rho_o ckd\underline{Q}}{4\pi r'}e^{-jkr'}$

b) The main radiation beam may be steered away from $\theta = 0$ when $\underline{\Gamma}(x) = \exp\{j\gamma x\}$. For this case, use the results of part a) to show

$$\underline{p}(r,\theta,t) = \frac{ja}{2r}\rho_o c U_o kL \exp\{j(\omega t - kr)\}\frac{\sin[L(\gamma + k\sin\theta)/2]}{L(\gamma + k\sin\theta)/2}.$$

You may find the following relationships useful

- $\int_a^b e^{j\beta x'}dx' = = \left[\dfrac{e^{j\beta x'}}{j\beta}\right]_a^b$ and,
- $\left[\dfrac{e^{jx} - e^{-jx}}{2j}\right] = \sin(x)$

NOTE: if you are running out of time, you may use the result of part b) to answer the subsequent questions c)-f)

c) What is the main beam angle, θ_{max}, where the radiated sound amplitude is maximum?

d) When $\gamma = 0$ (sometimes called "broadside") what is θ_{max}?

e) In terms of the product kL, find the full half-angle of the main beam = $\theta_{1/2} = |\theta_{max} - \theta_1|$, where θ_1 is the nodal direction closest to the main beam.

f) Are longer or shorter line sources better at radiating sound in narrow beams? (Only a very brief answer is needed)