PhD Qualifying Examination in Acoustics

Two-hours, closed-book

Examinees are permitted a one page crib sheet

Answer all parts of all three questions

1) A circular loudspeaker of diameter 12.5 cm is mounted in a rigid baffle and driven, in air, at a frequency of 6550 Hz. Assume the speed of sound is 343 m/s, and ρc= 400 Rayl.

a) Find the Rayleigh distance

b) How much acoustic power (mW) is radiated when the sound pressure level on axis is 90 dB at a distance of 1 m?

b) Find the half-power beamwidth (in the farfield)

d) Is there a pressure null on the axis of the loudspeaker? Explain your answer.

Work these problems as far as you can; clearly state all assumptions or approximations, and if possible, validate your approximations. You will receive full credit if your final expression(s) merely lack numerical evaluation.

$$p(x,y,z,t) = \frac{jk\rho_o c_o u_o e^{j\omega t}}{2\pi} \int_S \frac{e^{-jkR}}{R} dS$$

$$p(r,\theta,t) = \frac{ja\rho_o c_o u_o e^{j\omega t}}{r} \frac{ka}{2} e^{j(\omega t - kr)}$$

$$p = P_0 \left[e^{j(\omega t - kr)} - e^{j(\omega t - kr_1)} \right]$$

$$D(\theta) = \frac{2J_1(ka\sin\theta)}{ka\sin\theta}$$

$$\frac{2J_1(ka\sin\theta)}{ka\sin\theta} = \frac{1}{\sqrt{2}}$$

$$2\theta = 2\sin^{-1}(1.616/ka)$$

$$W = Su_{rms}^2 \operatorname{Re}(Z_p)$$

$$R_1(2ka) = 1 - \frac{2J_1(2ka)}{2ka}$$

$$W = \frac{\pi a^2}{2\rho c} P_0^2 R_1(2ka)$$

$$R_0 = S/\lambda = ka^2/2$$

$$p = \frac{P_0 R_0}{r} e^{j(\omega t - k(r - R_0))}$$

$$\sin\left(\pi a/\lambda \left(\sqrt{(a/r)^2 + 1} - (a/r) \right) \right) = 0$$

A6 TABLE OF DIRECTIVITIES AND IMPEDANCE FUNCTIONS FOR A PISTON

x	*Directivity Functions* ($x = ka \sin\theta$)		*Impedance Functions* ($x = 2ka$)	
	Pressure	*Intensity*	*Resistance*	*Resistance*
	$\frac{2J_1(x)}{x}$	$\left(\frac{2J_1(x)}{x}\right)^2$	$R_1(x)$	$X_1(x)$
0.0	1.0000	1.0000	0.0000	0.0000
0.2	0.9950	0.9900	0.0050	0.0847
0.4	0.9802	0.9608	0.0198	0.1680
0.6	0.9557	0.9134	0.0443	0.2486
0.8	0.9221	0.8503	0.0779	0.3253
1.0	0.8801	0.7746	0.1199	0.3969
1.2	0.8305	0.6897	0.1695	0.4624
1.4	0.7743	0.5995	0.2257	0.5207
1.6	0.7124	0.5075	0.2876	0.5713
1.8	0.6461	0.4174	0.3539	0.6134
2.0	0.5767	0.3326	0.4233	0.6468
2.2	0.5054	0.2554	0.4946	0.6711
2.4	0.4335	0.1879	0.5665	0.6862
2.6	0.3622	0.1326	0.6378	0.6925
2.8	0.2927	0.0857	0.7073	0.6903
3.0	0.2260	0.0511	0.7740	0.6800
3.2	0.1633	0.0267	0.8367	0.6623
3.4	0.1054	0.0111	0.8946	0.6381
3.6	0.0530	0.0028	0.9470	0.6081
3.8	+0.0068	0.00005	0.9932	0.5733
4.0	−0.0330	0.0011	1.0330	0.5349
4.5	−0.1027	0.0104	1.1027	0.4293
5.0	−0.1310	0.0172	1.1310	0.3232
5.5	−0.1242	0.0154	1.1242	0.2299
6.0	−0.0922	0.0085	1.0922	0.1594
6.5	−0.0473	0.0022	1.0473	0.1159
7.0	−0.0013	0.00000	1.0013	0.0989
7.5	+0.0361	0.0013	0.9639	0.1036
8.0	0.0587	0.0034	0.9413	0.1219
8.5	0.0643	0.0041	0.9357	0.1457
9.0	0.0545	0.0030	0.9455	0.1663
9.5	0.0339	0.0011	0.9661	0.1782
10.0	+0.0087	0.00008	0.9913	0.1784
10.5	−0.0150	0.0002	1.0150	0.1668
11.0	−0.0321	0.0010	1.0321	0.1464
11.5	−0.0397	0.0016	1.0397	0.1216
12.0	−0.0372	0.0014	1.0372	0.0973
12.5	−0.0265	0.0007	1.0265	0.0779
13.0	−0.0108	0.0001	1.0108	0.0662
13.5	+0.0056	0.00003	0.9944	0.0631
14.0	0.0191	0.0004	0.9809	0.0676
14.5	0.0267	0.0007	0.9733	0.0770
15.0	0.0273	0.0007	0.9727	0.0880
15.5	0.0216	0.0005	0.9784	0.0973
16.0	0.0113	0.0001	0.9887	0.1021

2) A layer of sound barrier that can be simplified and modeled as a layer of fluid with its acoustic impedance Z_2 and the speed of sound propagating in this material c_2 is inserted underwater. The speed of sound underwater is given by c_1 and the acoustic impedance of water is Z_1. For normal incident plane wave at frequency ω,

a) Derive the normal incidence plane-wave transmission and reflection coefficient when the layer thickness is *L*.

b) Derive the conditions that will result in total transmission through the layer. Give physical explanations why the total transmission through the barrier is achieved under the derived conditions.

c) Assuming that $Z_1 \ll Z_2$. Derive the conditions that will result in total reflection from the layer. Give physical explanations why the total reflection from the barrier is achieved under the derived conditions.

3) From a fluid mechanics viewpoint, dissipation due to friction at the side walls of a waveguide is a viscous effect associated with boundary layers. Viscosity also can cause dissipation within the entire field. A modification to the wave equation accounting for viscosity is obtained by adding a term accounting for shear stress in the momentum equation, thereby obtaining the Navier-Stokes equation. In a plane wave motion the viscous stresses are solely produced by molecules rubbing against each other as the fluid expands and contracts, and the corresponding effect is said to be bulk viscosity. The field equation accounting for this effect is

$$\frac{\partial^2 p}{\partial t^2} - \frac{\mu}{\rho_0}\Gamma\frac{\partial^3 p}{\partial x^2 \partial t} - c^2\frac{\partial^2 p}{\partial x^2} = 0$$

where μ is the dynamic viscosity (Pa-s) and Γ is a bulk viscosity coefficient, which is 4/3 for an ideal gas.

a) Determine the corresponding dispersion relation governing the one-dimensional propagation of a harmonic plane wave (that is the relationship between the wavenumber and the frequency).

b) Assuming the ratio μ/ρ_0 is small compared to c^2, linearize the previous relationship to determine the manner in which the phase speed and attenuation factor (related to the imaginary part of the wavenumber) depend on frequency.
HINT: $(1+x)^n \sim 1+nx$, when $x<<1$.