GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam – Spring Semester 2020

ACOUSTICS

EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

✷ Please sign your **name** on the back of this page —

PhD Qualifying Examination in Acoustics

Two-hours, closed-book

Examinees are permitted a one page crib sheet

Answer all parts of ALL three problems

PROBLEM 1

1. The wave equation for a spherically symmetric sound wave is the following

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

Where p represents the small amplitude fluctuations, r the distance from the origin, and c the speed of sound.

a. Re-express this equation as a function of derivatives of the product (pr) only, and thus determine its general solution. *[HINT: Multiply by r and consider that $(xf)_x = xf_x + f$)]*

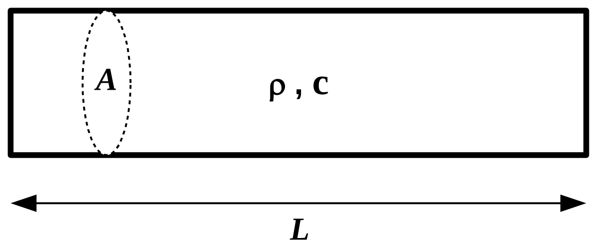
b. Show that only part of this general solution satisfies the Somerfield radiation condition, given by

$$\lim_{r\to\infty} r\left\{\frac{\partial p}{\partial t} + c\frac{\partial p}{\partial r}\right\} = 0$$

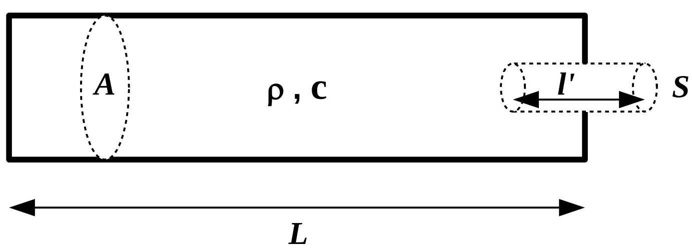
c. A pulsating sphere that radiates spherical waves into air, where the frequency is 100 Hz and the intensity 50 mW/m^2 at a distance of 1 m from the center of the sphere. What is the power radiated? What is the pressure amplitude and velocity amplitude at distance 50 cm from the center of the sphere?

PROBLEM 2

a) Consider the rigid closed tube with cross-sectional area *A* and length *L* as shown below and assume that the lateral dimensions are much smaller than the wavelengths of interest, i.e. plane pressure waves with associated volume velocity are propagating along the length of the tube. Derive the relationship for resonance frequencies of this tube. Plot the pressure and volume velocity distribution along the tube at the third resonance frequency.

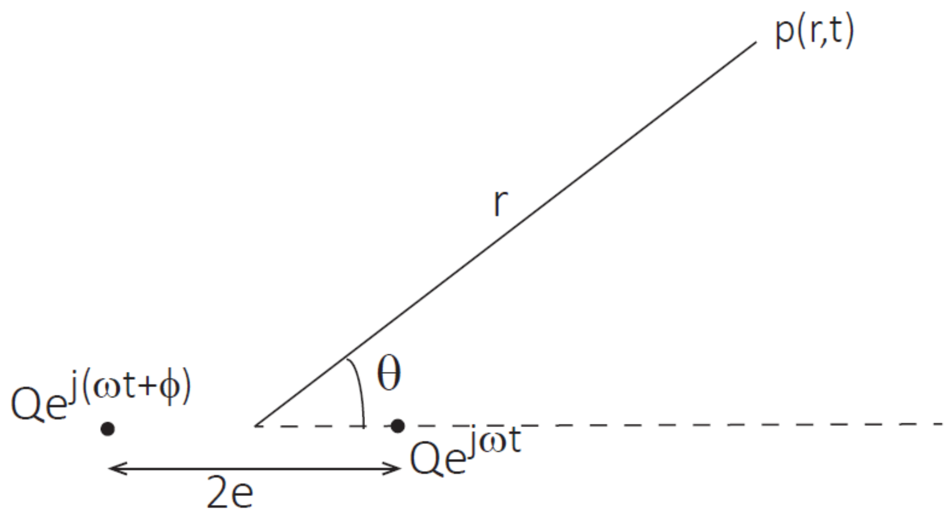


b) Find the relation for the resonance frequencies when a hole is drilled at one end of the tube as shown in the figure below. The effective length of the opening is *l'* and the cross-sectional area is *S*. Show that at very low frequencies this relation results in the classical Helmholtz resonator formula. Assume that there is no radiation loss at the opening, i.e. the impedance is purely reactive. Comment on the validity of the classical formula at higher frequencies.



PROBLEM 3

Consider two harmonic point sources separated by a distance 2e, as shown in the figure below. The phase difference between the volume velocity of the two sources is an angle ϕ



1. Derive the expression for the pressure radiated in the far field

2. Show that the magnitude of the pressure in the far field can be written as:

$$P(r) = \frac{1}{2}\rho_0 c \frac{Q}{\lambda r}\sqrt{2(1+\cos(ke\cos\theta + \phi))}$$

   Hint: the following trigonometric identities might be useful

$$\cos^2\theta + \sin^2\theta = 1$$
$$\cos(a+b) = \cos a\cos b - \sin a \sin b$$

3. Determine the nodal directions (that is the angles θ for which the pressure field is zero).