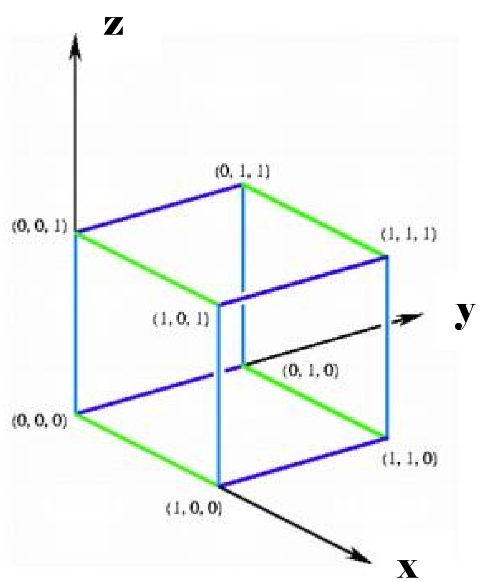
# APPLIED MATH WRITTEN EXAM

SPRING 2019

**1) Answer the following questions:**

**(a) Given the vector field** $\vec{v} = \dfrac{\left(x\,\hat{i} + y\,\hat{j} + z\,\hat{k}\right)}{R^3}$ **, where R is the magnitude of the vector** $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ **, determine the divergence of** $\vec{v}$**.**

**(b) Evaluate the flux of the vector field** $\vec{F} = x\,\hat{i} + \left(y + z\right)\hat{j} + 3z^2\,\hat{k}$ **over the surface of the unit cube shown below with corners at (0,0,0) and (1,1,1).**



**2) In view of least square errors, determine the values of "$a_1$" and "$a_2$" in** $y = a_1x_1 + a_2x_1x_2$ **that would best fit the following given data:**

| | | | |
|---|---|---|---|
| **$x_1$** | **1** | **2** | **3** |
| **$x_2$** | **1** | **2** | **3** |
| **y** | **5** | **4** | **3** |

**3) A college student owes \$1000 to a credit card company, which charges simple interest at an annual rate of 10%. The student makes payments continuously at a constant rate of \$10/month (\$120/year).**

**(a) Set up the initial value problem describing the situation.**
**(b) Solve the initial value problem of part (a).**
**(c) Find the time T it will take to pay off the debt.**

**4) Consider the following differential equation:** $3\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 4y = g(t)$

with $g(t) = 1 - u_\pi(t) = \begin{cases} 1 \; when \; 0 \le t < \pi \\ 0 \; when \; t \ge \pi \end{cases}$

and $u_\pi(t) = \begin{cases} 0 \; when \; 0 \le t < \pi \\ 1 \; when \; t \ge \pi \end{cases}$

Assume the initial conditions: $y(0) = c \;\; and \; \frac{dy(0)}{dt} = 0$

__Task__: find y(t)

__Hint__: Make use of the 2 following Laplace transformations (L=Laplace transform)

| $f(t) = L^{-1}\{F(s)\}$ | $F(s) = L\{f(t)\}$ |
|---|---|
| $e^{at}\sin(bt)$ | $\frac{b}{(s-a)^2 + b^2} \;, s > a$ |
| $e^{at}\cos(bt)$ | $\frac{s-a}{(s-a)^2 + b^2} \;, s > a$ |