COMPUTER-AIDED ENGINEERING
Ph.D. QUALIFIER EXAM – FALL 2018

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- All questions in this exam have a common theme: ***Hurricanes***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

Suppose that you are a design engineer in a water pump manufacturing company. Your current task is to design an impeller for a portable pump for the hurricane season. You want to use Bézier curves to model the profiles for both pressure and suction faces.

a) Write the analytical equation of the quadruple (the 4th order) Bézier curve.

b) You received the design of the first curve with the control points as
Curve 1:

(2, 20, 3)	(3, 21, 5)	(6, 22, 4)	(8, 23, 3)	(10, 22, 2)

What is the tangent vector (both direction and magnitude) at point (2, 20, 3)?

c) To design the second curve, you are given some of the control points as given below:
Curve 2:

(-7, 21, -5)	(-5, 22, -4)	(-3, 22, -4)	$\mathbf{P}_1$	(2, 20, 3)

To ensure the C^0 and C^1 continuity between the two curves, what are the coordinate values of $\mathbf{P}_1$? Explain how you decide the values in detail and show calculations.

d) You are given a third curve, Curve 3. You are asked to create a surface patch to model the interior of the pump by linearly lofting between Curve 1 and Curve 3. Explain the steps for linear lofting.

2) Finite-Element Analysis

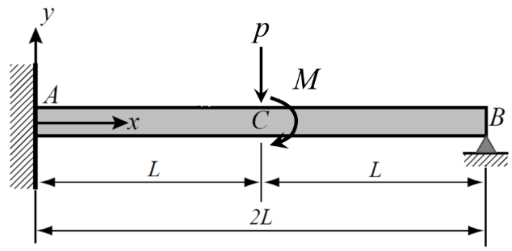


Figure 1

Recently, many roads and bridges in the Carolinas were damaged in a hurricane. You are asked to analyze one such bridge using the finite-element method. Assume that the external forces and moments can be simplified as a point force *p* and a bending moment *M* at the center of the bridge, as illustrated in Figure 1. Assume also that the bridge is clamped at one end and simply supported at the other end.

1. You are asked to determine the deflection at the center of the bridge *C*. Assume all missing material properties and geometry parameters to solve the problem.
    a. Select an appropriate type of element to analyze this problem, and state how many elements you would choose.
    b. Determine the individual matrix and the assembly matrix
    c. State the boundary conditions and loading conditions for all nodes for the problem. You need not solve for the deflection.
2. The FEA calculation of the deflection will be compared with the exact analytical solution. What would be the expected accuracy of the calculated deflection from the FEA compared to the analytical solution?
3. Assume that the right end *B* of the bridge is sitting on a other side of the bank where the ground acts as a spring with a spring constant *k*. State how your boundary and loading conditions will change for the right end *B*. There is no need to solve the problem.

<u>Element A - Stiffness Matrix</u>

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

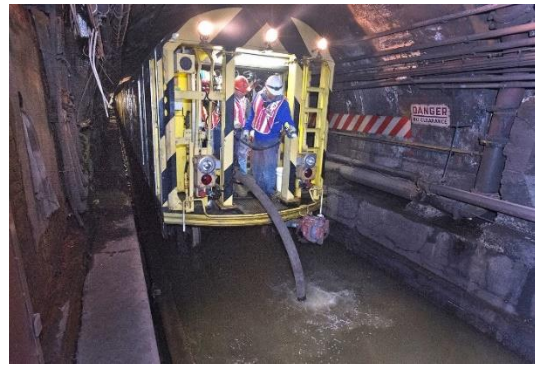
where *E*, *A*, and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines.

<u>Element B - Stiffness Matrix</u>

$$[K]=\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & -6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

where *E, I*, and *L* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively.

Question 3: Numerical Methods



During a hurricane season, a city subway system suffered from about 1 million gallons of water due to the rainfall, which must be pumped out from the underground on a daily basis. A perpetual hazard was that water could drip onto electrified equipment causing shorts.

The maintenance crew decided to use a variable speed pump to drain the flood to streets. The pumping rate *p* in terms of reducing the depth of water (inch/minute) was observed to depend on the duration of operation (T minute) and the revolutions per minute of the impeller (ω rpm).

An investigator reported the data as tabulated below:

Observations	*p* (in/min)	ω (rpm)	T (min)
1	1.1	100	25
2	1.35	200	18
3	2.18	150	36
4	1.78	180	20

(a) What type of numerical method would you use, if you want to develop a relationship of *p* with ω and *T*?

(b) According to the table above, describe the equation or algorithm you would use to predict *p*, given ω and T. Describe all necessary steps. However, it is not necessary to perform the actual computation.

(c) What are the steps to be taken to improve the accuracy of your prediction model? Explain your rationale and describe the procedure.