COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Spring 2015

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- All questions in this exam have a common theme: ***Nor'easter – Snow Storm***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

The monster snow storm has left behind some huge snow banks. Shown below is the cross-section of a snow bank that you need to remove with your shovel. Units are in meters.

The cross-section of the snow bank is modeled using a composite, cubic Bezier curve. The top surface of the snow bank is smooth, so the Bezier curves have C1 continuity. Control vertices are as follows: Curve 1: $\mathbf{p}_0 = (10, 0)$, $\mathbf{p}_1 = (9, 3)$, $\mathbf{p}_2 = (8, 4)$, $\mathbf{p}_3 = (6, 3)$; Curve 2: $\mathbf{q}_0 = (?, ?)$, $\mathbf{q}_1 = (?, ?)$, $\mathbf{q}_2 = (2, 4)$, $\mathbf{q}_3 = (0, 0)$.

Answer the following questions:

a) Determine the unknown control vertices $\mathbf{q}_0$ and $\mathbf{q}_1$.

b) Derive the equation for Curve 1. Simplify the equations into the form: $\mathbf{a}_3 u^3 + \mathbf{a}_2 u^2 + \mathbf{a}_1 u + \mathbf{a}_0 = \mathbf{p}(u)$

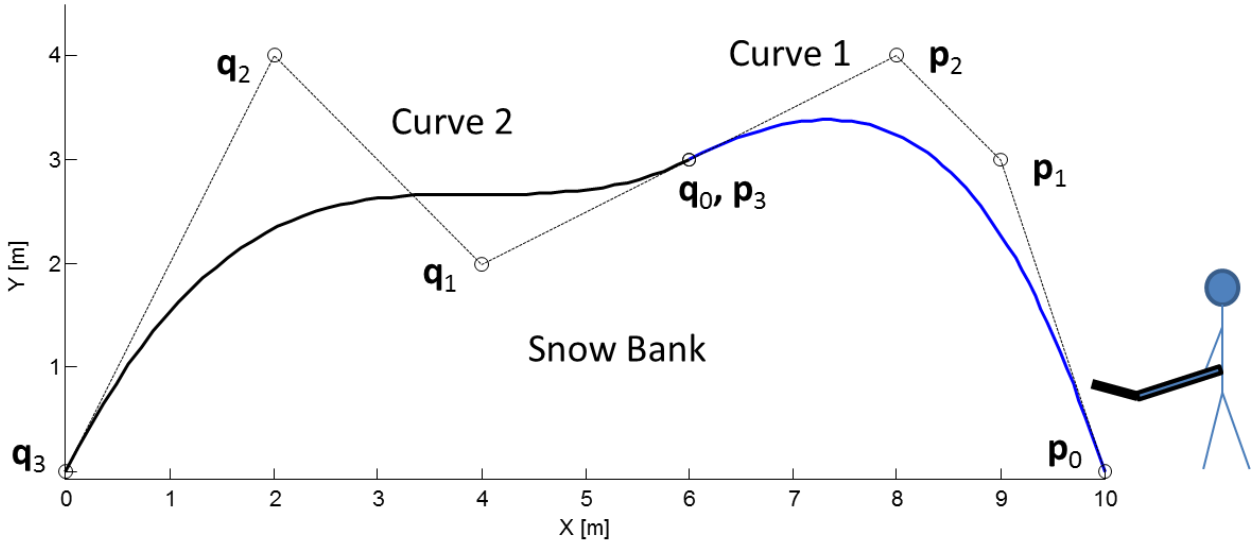
c) Compute the point on this first curve at $u = 0.3$.

The shoveling task appears daunting. You want to estimate the weight of the snow bank so that you know how much work is necessary. Assume the snow back is 20 meters long and it has a constant cross-section. Determine the snow bank volume that is represented by Curve 1.

d) Identify two different methods of computing the volume and outline how you would apply these methods to this problem.

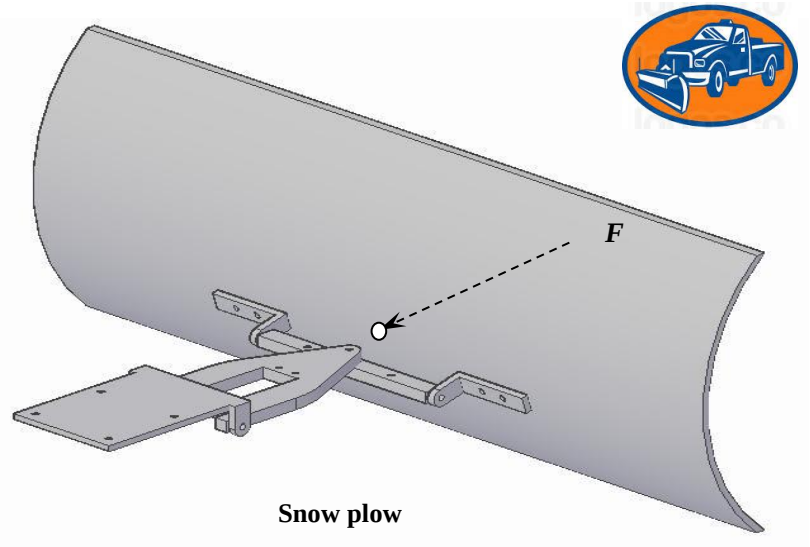
e) Select one of these methods. Set up the problem mathematically. Start on the solution process. You do not need to complete all of the calculations, but provide a start.

f) You start shoveling by inserting your shovel horizontally in the snow bank at a height of 1 m, near the beginning of Curve 1. You want to determine the weight of snow that you need to lift, assuming that your shovel is 0.4x0.3 m in size (that is, determine the volume of snow above the shovel after you insert it into the snow at a height of 1 m). Outline the steps needed to solve this problem.



Question 2 – Finite-Element Analysis

As a design engineer, you are assigned a task to design a snow plow. Finite-element analysis is applied to verify the design.



Snow plow

(a) The pressure from snow is simplified as a force (***F***) applied at the middle point. Build a finite-element model to estimate the deformation where the load ***F*** is applied. Make necessary assumptions of lengths, cross-section areas, materials, etc.

1. State all of your assumptions clearly.
2. Show all of your calculations.
3. Show the boundary conditions and loading conditions.
4. Write down the element stiffness matrix and assembly stiffness matrix.

(b) Discuss briefly how the model will be different if the pressure from snow is modeled as a distribution instead of a single force?

Element A - Stiffness Matrix

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; $l=(x_2-x_1)/L$ and $m=(y_2-y_1)/L$.

Element B - Stiffness Matrix

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}$$

where E, I, and L are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Question 3 – Numerical Analysis

The Northeast was hit by a series of winter storms in February. On one day, it began to snow at midnight and the snow continued to fall at a constant rate until noon. For instance, snow is falling at a constant rate of 1 in/hour and is melting at a rate equal to 1/4 of the amount already on the ground per hour. Thus, the thickness of the snow, ***x***, is measured in inches at time ***t*** hours.

(a) Write an equation for the thickness of the snow, ***x***.

(b) Estimate an approximation of how much snow fell between midnight and 1 a.m. The initial thickness of snow at midnight was 2 in. Use a step size of 0.5 hours.

(c) How would the accuracy of your estimate change if you use a step size of 0.25 hours? Answer this question without solving the problem.

At noon, snow stopped falling and the equilibrium level of snow was 4/3 in. A snowplow began clearing the snow from the streets. The speed of the snowplow, ***v***, has been monitored several times and is summarized in the table below.

t (sec)	0	2	4	6	8	10	...	...	...
v (in/sec)	10	9	8	9	8	7	...	...	...

(d) According to the table above, estimate an approximated volume of how much snow has been cleared up by the snowplow between 0 sec and 10 sec. Assume that the width of the plow, ***w*** is 40 in. Use the best strategy for attaining the highest accuracy of the estimation. Also, explain why you have selected the strategy.