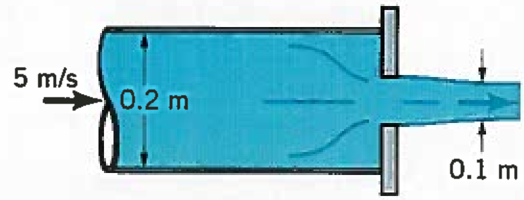
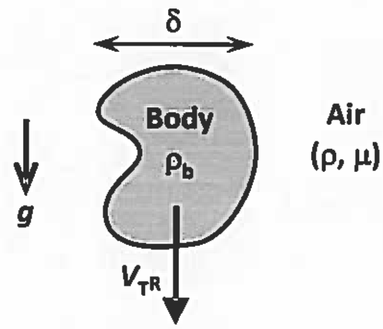
1. The exit plane of a 0.2-m diameter pipe is partially blocked by a plate with a hole in it that produces a 0.1-m diameter stream as shown in figure. The water velocity in the pipe is 5 m/s.



(a) If gravity and viscous effects are negligible, determine the force needed to hold the plate against the pipe.

(b) Explain if the pipe was vertical, how the effect of gravity would alter the results in (a) if the weight of the pipe and the plate could be neglected.

2. Consider a small body of density ρ_b with a characteristic lengthscale of δ falling through still air of density ρ and viscosity μ. If this flow is creeping, or Stokes, flow, the terminal (settling) velocity of the particle V_T is only a function of δ, μ, the gravitational acceleration g, and the density difference between the body and the air $\Delta\rho \equiv \rho_b - \rho$.



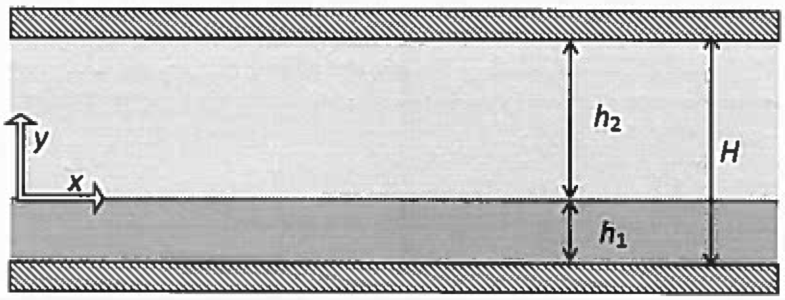
(a) Use dimensional analysis to determine how V_T depends upon these other parameters.

(b) The aerodynamic drag on a small body in creeping flow D is known to be a function of its speed U, its characteristic lengthscale δ, and the viscosity of the air μ. Use dimensional analysis to determine how D depends upon this set of parameters.

(c) When a small body achieves its terminal velocity (so $U = V_T$), the aerodynamic drag D will be balanced by the "differential weight" of the body, or the difference between the weight of the body and the buoyancy force on the body. If this differential weight is proportional to $\Delta\rho$, use your results from part (b) to "improve" your result for part (a) by determining a functional relationship between the dimensionless groups (*i.e.*, Π terms) that you found in part (a).

3. Consider two horizontally layered, immiscible, incompressible fluids flowing in a steady laminar manner between to fixed infinite parallel plates a distance, *H*, apart as shown in the figure. The flow is driven by a constant pressure gradient, *dP/dx*. The lower fluid with density, ρ_1, and viscosity, μ_1, has a height, h_1. The upper fluid with density, ρ_2, and viscosity, μ_2, has a height, h_2. Gravity acts in the negative *y* direction.

$h_1 + h_2 = H$
$h_1 \neq h_2$
$\rho_2 < \rho_1$
$dP/dx = const < 0$
$\mu_1 \neq \mu_2$



(a) Obtain expressions for the velocity profiles in the lower liquid, u_1, and upper liquid, u_2, as a function of position (x,y,z) and the problem parameters $(\mu_1, \rho_1, \mu_2, \rho_2, dP/dx, h_1, h_2)$.

(b) Sketch qualitatively the velocity profiles for the cases of $h_2/h_1 = 2$ and 1) air over water, and 2) oil over water.

Fluid	μ (N·s/m^2)	ρ (kg/m^3)
Oil	$1 \cdot 10^{-1}$	900
Water	$1 \cdot 10^{-3}$	1000
Air	$2 \cdot 10^{-5}$	1.2

(c) Comment on stability of the flow for the following conditions: 1) $\rho_1 = \rho_2, \mu_1 = \mu_2$; 2) $\rho_1 < \rho_2, \mu_1 = \mu_2$; 3) $\rho_1 > \rho_2, \mu_1 = \mu_2$.

4. A hinged rectangular massless gate (measuring a x b) is used to separate between two adjacent reservoirs A and B holding liquids of densities ρ_A and $\rho_B = 0.5\rho_A$ as shown in Figure 4a below.

a) If $h_A = a/2$, determine h_B so that the hydrostatic forces on both sides of the gate are equal. Determine the line of action of each force as measured from the hinge.

b) In order to keep the gate closed, it is proposed to attach a counterweight having volume $V = (a/2)^2 b$ and weight W to the gate as shown in Figure 4b. The immersed counterweight is attached to the gate using a thin (massless) plate such that the distance between the centroid of W and the gate is a as shown below. Determine the minimum W that is needed to keep the gate in part (a) closed.

Note: For a rectangle in the x-y plane measuring c x d, where c and d are collinear with the x and y axes, respectively, $I_{xc} = (d)(c)^3/12$.

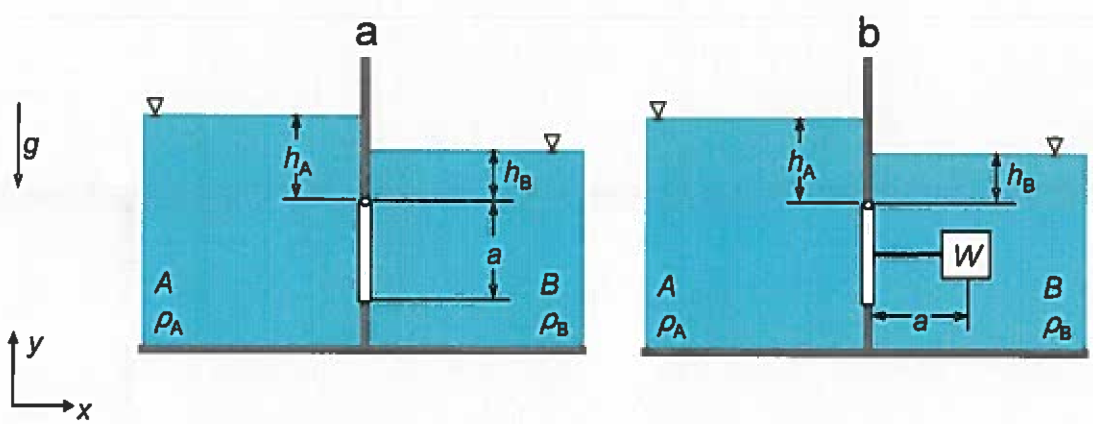


Figure 4