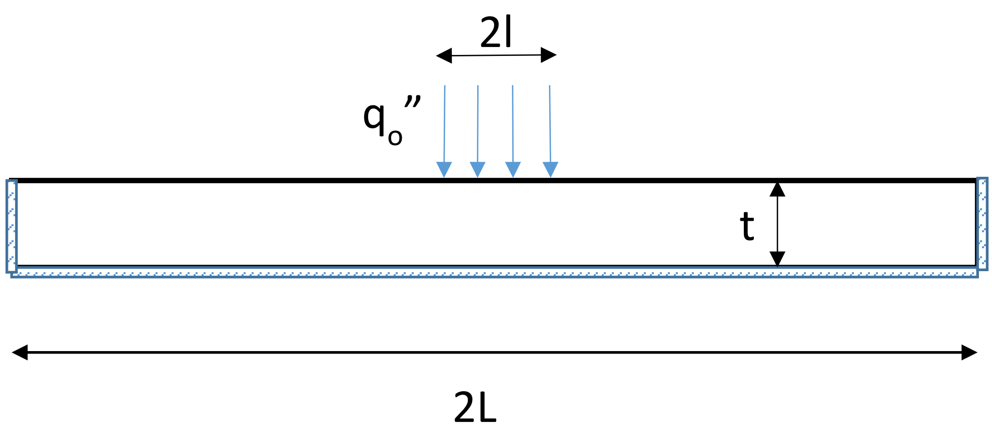
PhD Qualifying Exam
Heat Transfer, Written Exam

Problem 1

A thin plate of thickness t, length 2L, width W perpendicular to the plane shown (with W>>L), and thermal conductivity k is exposed centrally to a heat flux of q_o" over a length 2l. Outside of the heat flux region, the plate is exposed to a convective cooling environment at T_f, with a convection coefficient h. The sides and bottom of the plate are insulated. Assuming no through-thickness variation in plate temperature:

1. (40%) Derive an equation and identify appropriate conditions needed to determine the steady state temperature distribution along the length of the flat plate beyond the region of application of the heat flux.
2. (30%) Solve the governing equation to determine the temperature distribution.
3. (30%) If the plate temperature is measured at the end of the heat input region, show how the thermal conductivity of the plate can be determined.

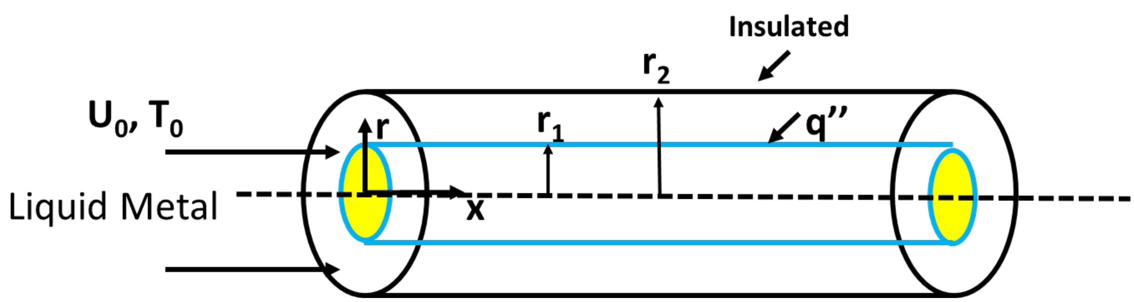


Problem 2

A liquid metal is flowing through an annular tube of inner radius r_1 and outer radius r_2. The temperature of liquid metal at the inlet of tube is T_o and flow profile at inlet is uniform with velocity U_o. The flow is laminar; axial conduction and viscous dissipation can be neglected, and fluid properties such as thermal conductivity k, density ρ, viscosity μ , and heat capacity C_p can be assumed constant. The inner surface of tube is subjected to constant heat flux **q''** and outer surface is insulated.

a) Consider a cross-section at a distance x from the inlet of tube, where hydrodynamic boundary layer is thin but flow is thermally fully developed. Plot the velocity and temperature distribution at this location as a function of radius. Write appropriate equations which can be used to find temperature as a function of radius T(r), list your assumptions, and derive an expression for T(r). (50 %)

b) Find local $Nu_i = hD_h/k$ at the inner surface of tube at the cross-section investigated in part (a). Here D_h is hydraulic diameter and h is convective heat transfer coefficient. (50 %)



Problem 3

Problem #3: Consider a composite roof for sustainable house (Fig. 1), which combines the solar water heater WH (to capture solar irradiation for heating water) and the thermal radiator TR (to reduce a cooling load on the HVAC system during the hot summer days). Given two material options "a" and "b" whose spectral directional emissivity is specified (Fig. 2), select the best material for making the WH and TR. To support your material selection, consider the case of clear day and when the noon sun is straight overhead (see Fig. 1). Use an energy balance along with the appropriate surface radiative properties to explain your selection. Do not perform any calculations.

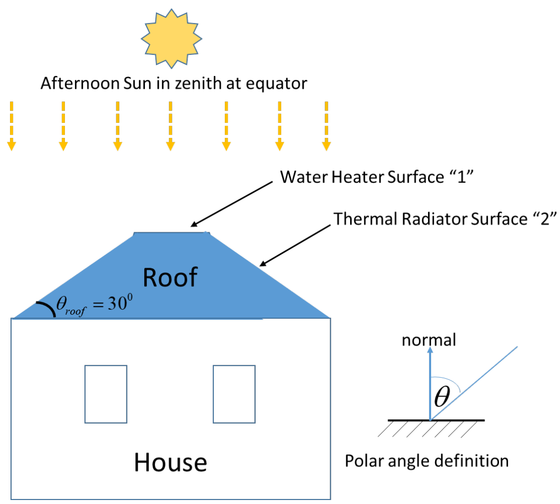


Fig. 1: Schematic of the house/roof subject to solar irradiation in the afternoon, including definition of the polar angle to be used in performing analysis of the problem.

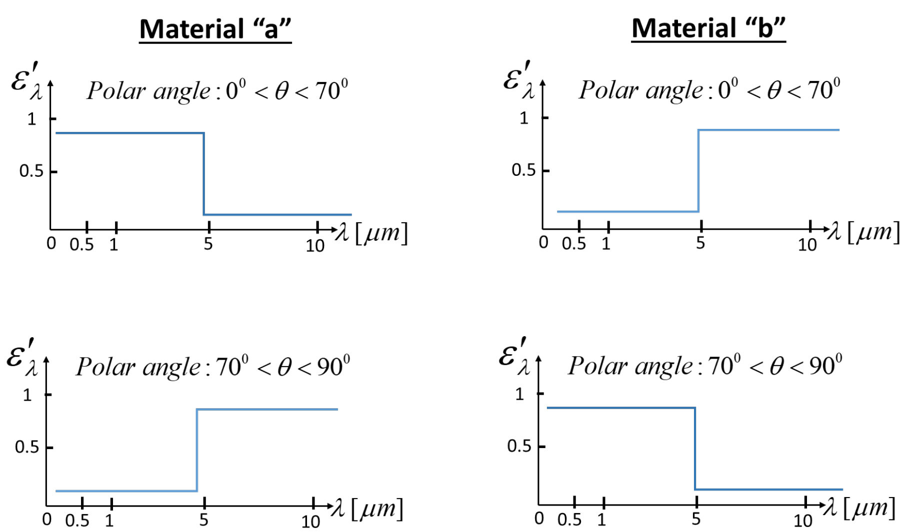


Fig. 2: Spectral directional emissivity of two material options available for construction of WH and TR.

Useful facts: Sun can be treated as a black body at ~5778K with the direct-normal component of solar irradiation as the principal source of radiative heating. Wien's Displacement Law

$\left(n\lambda T\right)_{\max} = 2898\ \mu\mathrm{m}\cdot\mathrm{K}$ provides the wavelength of maximum of the spectral blackbody emissive power for the black body at temperature T radiating out into a medium with refractive index *n*.