GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam – Spring Semester 2020

__________HEAT TRANSFER_________

EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

✷ Please sign your **name** on the back of this page —

PhD Qualifying Exam
Heat Transfer, Written Exam
Spring 2020

Problem 1

The front of eyeball can be considered as a hemisphere of radius r_2 (see schematic in Figure 1). The inner and outer radii of cornea are r_1 and r_2, respectively. The anterior chamber behind the cornea is filled with a viscous liquid of thermal conductivity k_l, which is half of the thermal conductivity of the tissues of cornea, k_{co} (k_l=0.5* k_{co}). A chip is implanted at the center of hemispherical eyeball to better assist in vision of defective eye. The chip is also of hemispherical shape of radius r_0 with same center as eyeball. The volumetric heat dissipation in chip is uniform and equal to $\dot{q}$. The thermal conductivity of chip k_{ch} is 5 times larger than the thermal conductivity of viscous liquid (k_{ch}= 5* k_l). The outer surface of cornea is exposed to ambient air at temperature T_∞ and convective heat transfer coefficient *h*. The chip manufacturer needs to confirm that maximum temperature of the chip does not exceed T_{th}. Assume constant properties, steady state and static viscous liquid in anterior chamber. The flat surface of the hemispherical eyeball can be considered insulated (*Hint: Carefully consider the boundary conditions to simplify the problem*).

a) Plot temperature along the radial direction from the center of chip to the center of cornea (dashed line in figure below). Describe the major characteristics of this plot considering the conductivity values specified. (3 points)
b) Identify maximum value of $\dot{q}$ for which peak temperature in chip will not exceed T_{th}? What is corresponding temperature at the outer surface of cornea? (7 points)

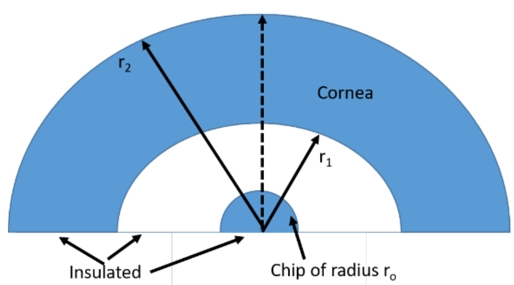

Figure 1.

Hint: 3D Heat Conduction Equation in Spherical coordinates is given as:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(k\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(k\frac{\partial T}{\partial\phi}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

Problem 2

For a flow through a pipe of diameter D with a constant wall heat flux, the fully developed velocity and temperature profiles are both found to be parabolic. Find the value of the Nusselt number for this flow. **(7 points)**

How would you compare (qualitatively, no detailed derivations are required) this value with the case of liquid metal flow. Compare and comment based on the respective velocity and temperature profiles. **(3 points)**

Problem 3

Consider two very large in extent, parallel, gray, diffuse, opaque plates (each of area A) with emissivities ε_1 and ε_2 held at uniform temperatures of T_1 and T_2, respectively, and separated by vacuum.

1. Derive the closed-form analytical expression for net rate of radiative heat exchange Q_{12} between the plates.
2. Based on your derived expression in part 1, discuss the requirement on surface emissivitities to minimize the heat transfer rate between two surfaces.
3. Use the results from part 1 to derive an expression for net rate of radiative heat exchange Q_{12} if one inserts a very thin, opaque planar plate 3 in between of plates 1 and 2. You can assume an equal emissivity on both sides of plate 3. *Hint: Use a radiation resistance between 2 surfaces and the overall resistance network for the system.*
4. From the expression derived in part 3, discuss the requirement on the surface radiative properties of plate 3 to minimize the net rate of radiative heat exchange Q_{12} between plates 1 and 2.

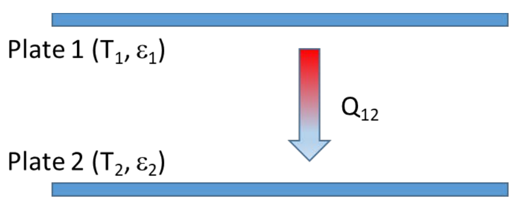

Figure for Parts 1 and 2

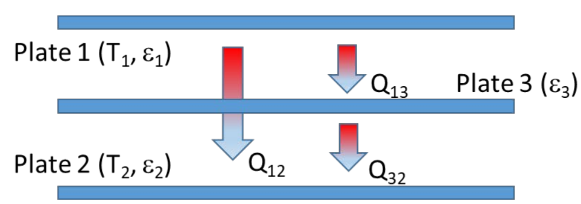

Figure for Parts 3 and 4