GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff

School of Mechanical Engineering

Ph.D. Qualifiers Exam – Spring Semester 2020

PLEASE WORK THREE OF THE FOUR PROBLEMS

__ SYSTEM DYNAMICS & CONTROL

EXAM AREA

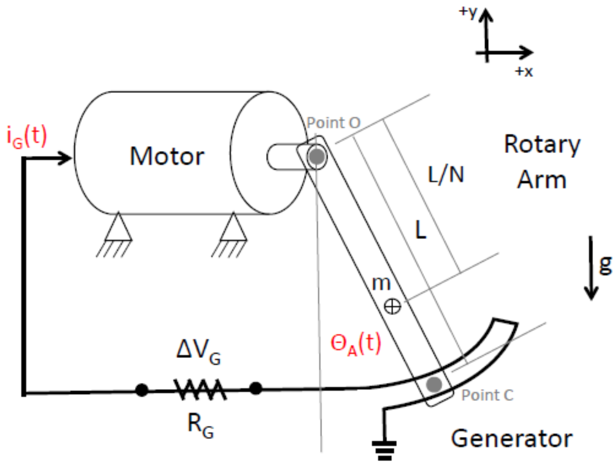
Assigned Number (DO NOT SIGN YOUR NAME)

✷ Please sign your **name** on the back of this page —

Dynamics Systems & Control Ph.D. Qualifying Exam
Fall 2019

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the 3 problems that you select, show all your work in order to receive proper credit. You are allowed to use a scientific, non-graphing calculator. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1: Consider the electromechanical system shown below and answer the questions.



A Rotary Arm with Mass m; length L; and center of mass location L/N attaches a motor to a Generator at Point O. The Motor produces torque $T_M(t)$ proportional to Generator current $i_G(t)$ according to:

$T_M(t) = K_M\, i_G(t)$ (Eq. 1)

The Generator produces current $i_G(t)$ proportional to the velocity of the Rotary Arm contact Point C according to:

$i_G(t) = K_G\ V_C$ (Eq. 2)

The Generator produces a resistance force $F_G(t)$ tangential to the Rotary Arm contact Point C that is proportional to the voltage drop $\Delta V_G(t)$ across a resistor with resistance R_G according to:

$F_G(t) = K_F\ \Delta V_G(t)$ (Eq. 3)

-Assume gravity acts.

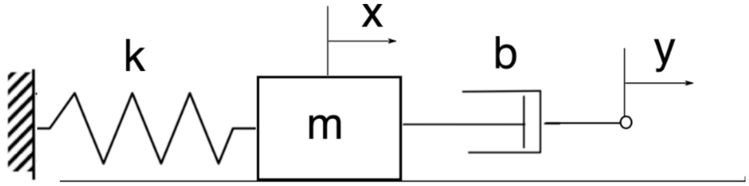
-Assume the moment of inertia of the Rotary Arm: $I = m\,(L/N)^2$

Questions:

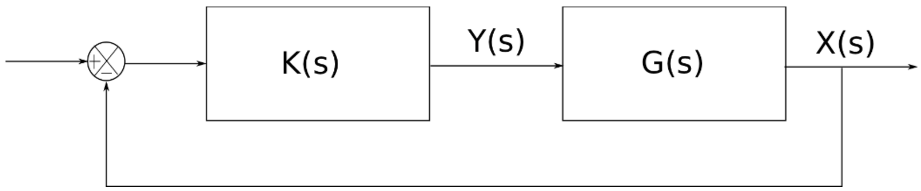
1) Obtain the equations of motion for the Rotary Arm. $\ddot{\theta}_A(t) = ?$
2) Linearize the equations of motion about $\theta_A(0), \dot{\theta}_A(0) = 0$ and express them in state-space form $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & \end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$. What is the characteristic equation?
3) Are there parameter settings (m, L, N, R_G, K_M, K_G, K_F) that make the system unstable? If so describe using an algebraic expression.
4) Explain what happens to the system response as N: $\infty \rightarrow 1$ for m, L, N, R_G, K_M, K_G, $K_F = 1$ using a sketch of the time response $\theta_A(t)$ with $\theta_A(0) = \frac{\pi}{2}$ and $\dot{\theta}_A(0) = 0$.

Answer Sheet for Problem 1

Problem 2: Consider the mechanical system shown in the figure. Displacements x and y denote the position of the mass and the right-hand-side position of the damper, respectively.

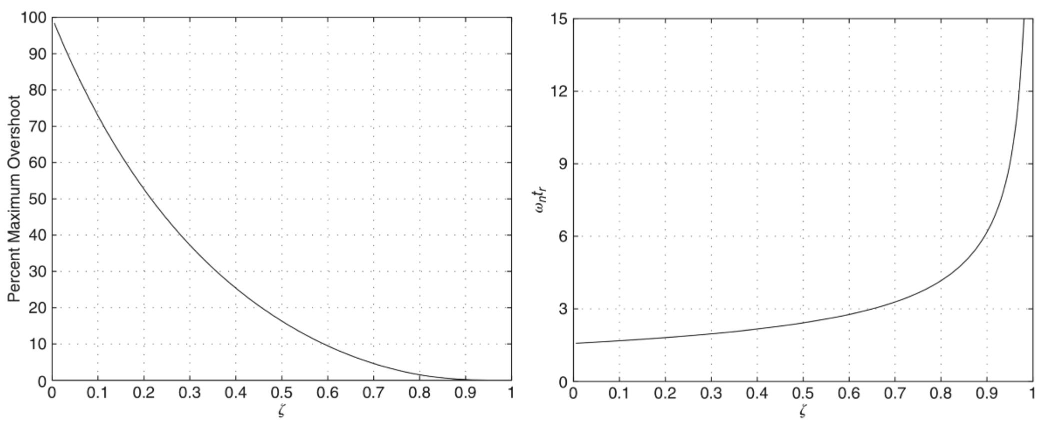


1) Derive the system dynamic equation and transfer function $X(s)/Y(s)$.
2) Define $G(s) = X(s)/Y(s)$ and assume a PI controller $K(s) = K_p + K_i/s$, where K_p and K_i are positive. Given the following feedback control diagram, please use Routh's method (i.e., the Routh stability table) to determine under what condition the closed-loop system is stable.



For problems (3) and (4), you only need to answer either one of them. You still need to answer (5).

3) Assume $m = 1, b = 4, k = 8$ and input y(t) is a unit step function, compute the steady-state value of x(t), maximum percent overshoot, the maximum overshoot, the rise time t_r. [Hint: use the two transient response parameter figures below]

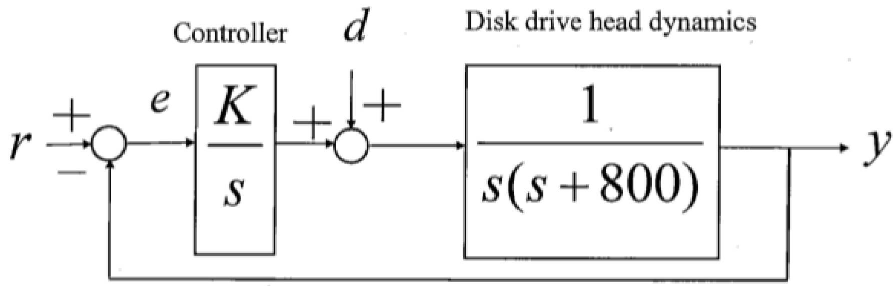


4) Assume $m = 9, k = 4$. Set the value of the damping constant b so that both of the following specifications are satisfied. Give priority to the overshoot specification. If both cannot be satisfied, state the reason. (i) Maximum percent overshoot should be as small as possible and no greater than 20%. (ii) 100% rise time should be as small as possible and no greater than 3 s. [Hint: also use the two transient response figures above]

5) Reconsider $X(s)/Y(s)$ derived in part (a). Derive the fact that the peak time is the same for all characteristic roots having the same imaginary part.

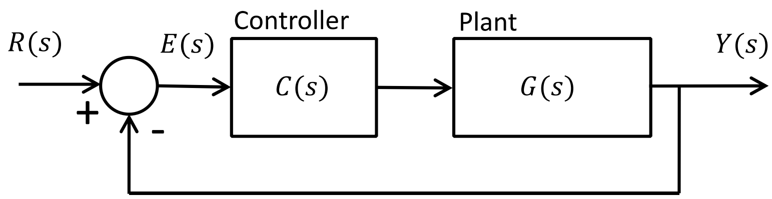
Answer Sheet for Problem 2

Problem 3: The block diagram below depicts a disk drive head position control system.

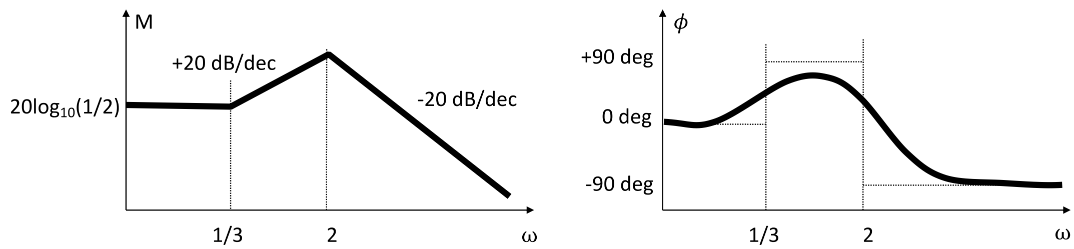


1) Sketch the root locus and discuss the stability with any positive gain K (when d=0).
2) If we replace the integral controller with a proportional-integral controller $\frac{K(s+b)}{s}$, then what values of b will the closed-loop system be stable for all positive values of K (when d=0)?
3) Choose a value of b for which the system will be stable for all positive K, and sketch the root locus for this system (when d=0).
4) What is the steady-state error for a step reference assuming d=0?
5) What is the steady-state error for a step disturbance assuming r=0?

Problem 4: You've been given a plant of the form $(s) = \frac{1}{s^2 - as + b}$, where a and b are positive constants. However, you do not know how to identify any of the constants because the system is unstable



1) Your professor suggests that you close the loop to make the system stable, identify the closed loop dynamics, and try to back calculate your constants. You were able to stabilize the system and generate a **closed loop bode plot** of $Y(s)/R(s)$ (shown below). What is the steady state error to a unit step for this closed loop system?



2) The closed loop bode plot in the previous part was constructed with the controller $C(s) = K_p(1 + K_d s)$ with parameter values $K_p = 2$ and $K_d = 3$. You purposefully tuned your controller so that the effective damping is $\zeta = 1$. Use this information to solve for your original system constants a and b.
3) Sketch a rough response to a unit step for the closed loop system from parts c) and d). What is the initial slope and steady state value? In addition, sketch the step response of the same system if no zero was present (with the same pole locations and same DC gain). How do the two step responses compare? Does either system oscillate?
4) Draw the frequency response for the original plant (G(s)).
5) You have to change the sensor from an ideal one to one with model (1/(5s +1)). Draw the Bode Plot for the open loop system. Provide a qualitative discussion on how sensor dynamics affect closed loop performance.

Answer Sheet for Problem 4