PLEASE NOTE:

You need to correctly answer only 3 out of the 4 problems to receive full credit. In case you attempt all 4 problems, clearly state which 3 problems you want to be graded. If you do not explicitly so indicate, the 3 lowest scores will be used.

Problem #1

A frame *ABC* is loaded at point *C* by a force *P* acting at an angle α to the horizontal (see Figure 1). Both members of the frame have the same length *L*, same moment of inertia *I*, and the same Young's modulus *E*. Determine the angle α so that the deflection of point *C* is in the same direction as the load. (**Disregard the effects of axial deformations and consider only the effects of bending due to the load *P*. You may use the beam bending equations given at the end.**)

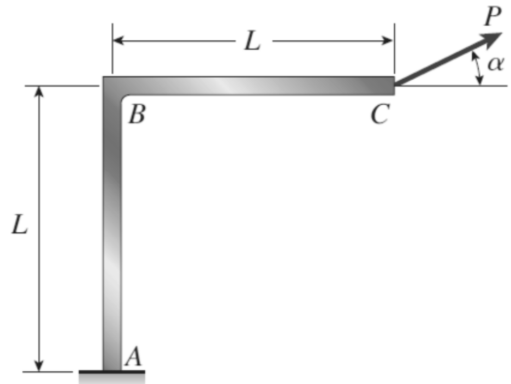


Figure 1

Problem #2

A rectangular beam has c = 40 mm (2c = 80 mm) and d = 25 mm. This beam is made from a metal that exhibits elastic-perfectly plastic behavior. The Young's modulus is E = 120 GPa, and the yield stress is σ_Y = 450 MPa. A bending moment of M = 15,000 N-m is applied as shown in the figure below, and then this moment is removed. Assume that the beam is long (along the x-direction) compared to its thickness.

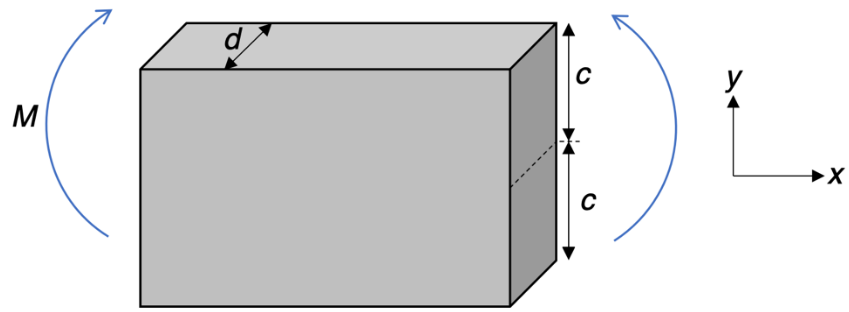


Figure 2

(a) Determine the thickness of the elastic core under the applied bending moment.

(b) Plot the stress distribution through the cross section of the beam under loading (moment M) as a function of distance y from the neutral axis (from y = -c to y = c).

(c) Determine the residual stresses after removal of the load and plot their distribution as a function of distance y from the neutral axis (from y = -c to y = c).

(d) Plot the stress-strain path for the upper surface of the beam during this process, and find and label the strain at the yield point.

(e) Consider the final stress state of a point on the top surface of the beam after unloading. Write out the full stress tensor, and find the maximum resolved shear stress σ_{xy}.

Problem #3

A symmetric three-bar planar truss in Figure 3a is loaded by a single horizontal force *P* at joint *A*. The members of the truss are all made of the same linearly elastic, perfectly plastic material (see Figure 3b) with $\sigma_y = 36$ ksi and $E = 30\times10^3$ ksi, and all have a cross-sectional area $A = 2.0$ in^2.

(a) Determine the load P_y at which first yielding occurs, and determine the corresponding displacement u_y of joint *A*, where the load *P* is applied.

(b) Determine the load P_u at which yielding occurs in the remaining member(s) of the truss, and determine the corresponding displacement u_u of joint *A*.

(c) Sketch a load-displacement diagram, that is, sketch a diagram of *P* versus *u* up to P_u.

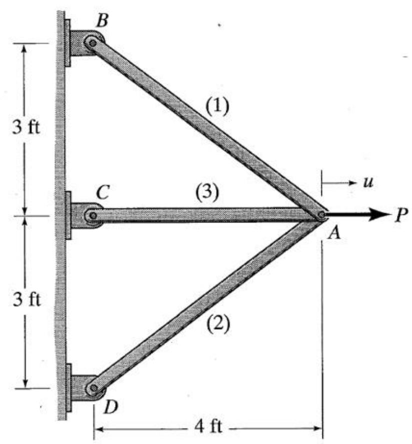


Figure 3a

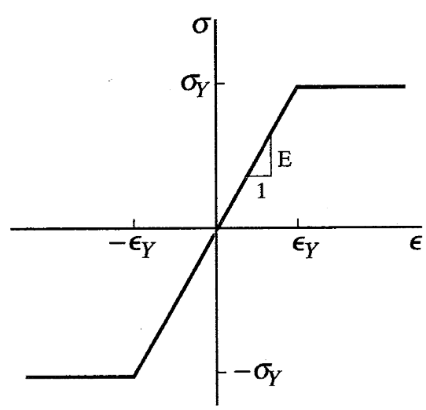


Figure 3b

Problem #4

A cylindrical steel pressure vessel of 7.5 m diameter and 40 mm wall thickness (Figure 4) is to operate at working pressure of 5.1 MPa. The fracture toughness of the steel is 200 $MPa\sqrt{m}$. The growth of the crack by fatigue may be represented approximately by the equation $da/dN = A(\Delta K)^4$, where $A = 2.44 \times 10^{-14} (MPa)^{-4} m^{-1}$. The design assumes that failure will take place by fast fracture from a crack which as extended gradually along the length of the vessel by fatigue. To prevent fast fracture, the total number of loading cycles from zero to full load and back to zero again must not exceed 3000.

Figure 4

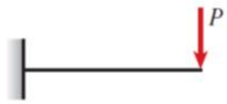
(a) Identify whether it is the axial stress or the hoop stress that is responsible for the failure and derive the expression to relate that stress to working pressure.

(b) Given the fracture toughness $K = \sigma\sqrt{\pi a/2}$, where σ is the stress and a is the crack length, calculate the maximum allowable initial defect length in the structure.

(c) Find the minimum pressure to which the vessel must be tested before use to guarantee against fatigue failure in under the 3000 load cycles.

Formulas of Beam Bending

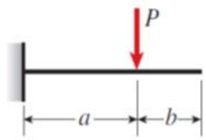
1



$$v = -\frac{Px^2}{6EI}(3L - x) \qquad v' = -\frac{Px}{2EI}(2L - x)$$

$$\delta_B = \frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$$

2



$$v = -\frac{Px^2}{6EI}(3a - x) \qquad v' = -\frac{Px}{2EI}(2a - x) \qquad (0 \le x \le a)$$

$$v = -\frac{Pa^2}{6EI}(3x - a) \qquad v' = -\frac{Pa^2}{2EI} \qquad (a \le x \le L)$$

$$\text{At } x = a\text{:} \quad v = -\frac{Pa^3}{3EI} \qquad v' = -\frac{Pa^2}{2EI}$$

$$\delta_B = \frac{Pa^2}{6EI}(3L - a) \qquad \theta_B = \frac{Pa^2}{2EI}$$

3

$$v = -\frac{M_0 x^2}{2EI} \qquad v' = -\frac{M_0 x}{EI}$$

$$\delta_B = \frac{M_0 L^2}{2EI} \qquad \theta_B = \frac{M_0 L}{EI}$$

4

$$v = -\frac{M_0 x}{6LEI}(2L^2 - 3Lx + x^2) \qquad v' = -\frac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$$

$$\delta_C = \frac{M_0 L^2}{16EI} \qquad \theta_A = \frac{M_0 L}{3EI} \qquad \theta_B = \frac{M_0 L}{6EI}$$

$$x_1 = L\left(1 - \frac{\sqrt{3}}{3}\right) \quad \text{and} \quad \delta_{\max} = \frac{M_0 L^2}{9\sqrt{3}EI}$$

5

$$v = -\frac{M_0 x}{2EI}(L - x) \qquad v' = -\frac{M_0}{2EI}(L - 2x)$$

$$\delta_C = \delta_{\max} = \frac{M_0 L^2}{8EI} \qquad \theta_A = \theta_B = \frac{M_0 L}{2EI}$$