System Dynamics & Controls Qualifier Fall 2012

Work all 4 questions.

Q1:

Consider a linear single-input single-output dynamic system whose transfer function is represented as $G(s) = \frac{s+a}{(s+b)(s+c)}$, $(a, b, c$: constants). The following properties are known about $G(s)$ and its unity feedback system.

Property 1: $G(s)$ has one zero at 2 and two stable poles.

Property 2: The unit step response of $G(s)$ is oscillatory with a damping factor of $\frac{1}{\sqrt{2}} \cong 0.707$.

Property 3: The unity feedback system is marginally stable

(1) Determine $G(s)$.

(2) Determine *all* of the closed loop poles of the unity feedback system. Sketch the root-locus plot (a general sketch without specifying break-in/out points or angles of departure/arrival is OK) and show the obtained closed-loop poles.

Q2:

Consider a unity-feedback system with the open-loop (feedforward) transfer function as

$$G(s) = \frac{K}{(s+5)(s+20)(s+50)}.$$

(1) Consider $K = 1$. Sketch the Bode diagram (No need to be accurate. Just a simple sketch.)

(2) First, evaluate the frequency as the phase angle of $G(s)$ is -180^0. Next, use this to determine the gain in dB as $K = 1$. Finally, use the gain to determine the range of K for the stability of the closed-loop system (assume $K > 0$).

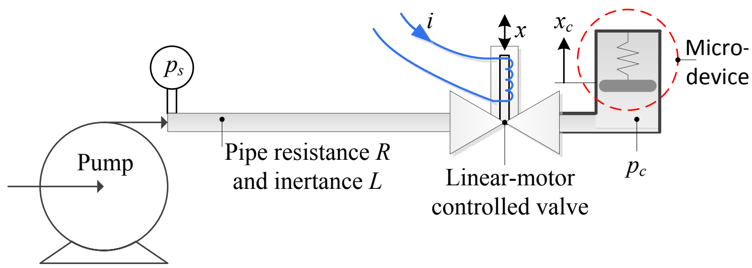
(3) Suppose the input to the feedback system is $A \sin \omega t$. Show the analytical expression of the steady-state output for $K = 1$.

Q3:

A motor has the transfer function 1/(s+1). Design a controller so the closed-loop system shows an underdamped response that reaches steady-state in a single period of oscillation of 0.1sec.

Q4:

Consider the schematics shown below illustrating a liquid supply system for controlling the pressure p_c of a micro-device which can be modeled as a mass-less piston (with area A) against a spring-damper (k_c and b_c) system. The liquid at pressure p_s flows through a variable valve (with flowrate $q = C_d A_o x\sqrt{\Delta p}$ where $C_d A_o$ is a constant; and $0 \le x \le 1$ is a displacement) to the micro-device. The valve is actuated by a permanent-magnet linear-motor which generates an electromagnetic force linearly proportional to the product of the displacement x and the controlling current i).



For high-bandwidth design, the rotor dynamics of the linear-motor (which can be modeled as a translational mass-damper-spring system) and the pipe inertance must be taken into account.

Derive the equation of motion using the Laplace transform; specifically, express the controlled pressure $P_c(s)$ in terms of supply pressure $P_s(s)$ and controlling current $I(s)$ about a steady-state operating point. Be sure to define the operating point at which your linear approximation is derived. (Note if short on time it is not necessary to derive this final equation but briefly explain how it can be obtained from a set of equations.)