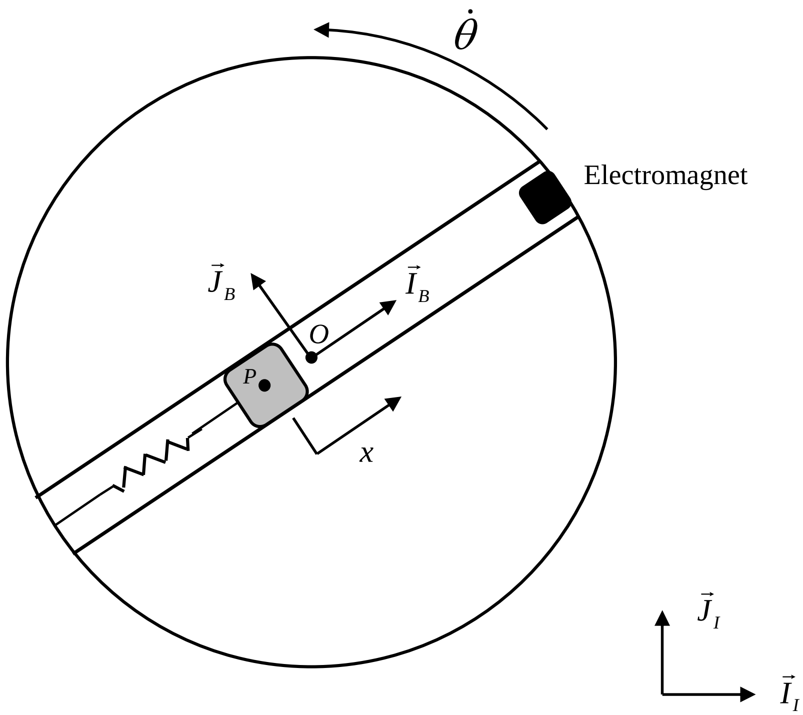
Dynamics Systems & Control Ph.D. Qualifying Exam
Spring 2016

Instructions:

Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the 3 problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.
Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1

A turntable of radius R rotates in the $\vec{I}_I - \vec{J}_I$ plane as shown with a constant angular speed $\dot{\theta}$. A slot is cut in the turntable, and a slider of mass m slides within the slot. The variable x measures the position of the slider in the slot, where $x = 0$ denotes that point P is located at the center of the turntable O. The slider is connected to a spring of unstretched length R (same as the radius of the turntable). Furthermore, the slider is subject to a frictional force exerted by the walls of the slot. Assume this friction force can be modeled as viscous friction with a damping coefficient d (i.e., the frictional force along $\vec{I}_B$ is given by $-d\dot{x}$). The slider is magnetic, and is controlled via an electromagnet at one end of the slot which exerts a magnetic force on the slider u in the $\vec{I}_B$ direction. Note that frame B rotates with the turntable and is centered at O, while frame I is an inertial reference frame. You may consider the slider to be a point mass for this problem.



a) Find the transfer function $G(s) = X(s)/U(s)$ from the control input to the position of the slider.

b) Suppose you are designing this system and considering two possible sets of design parameters:

Design A:	Design B:
$\dot{\theta} = 20$ rad/sec	$\dot{\theta} = 10$ rad/sec
m = 2 kg	m = 2 kg
k = 1000 N/m	k = 200 N/m
d = 5 N/(m/s)	d = 4 N/(m/s)
R = 1 m	R = 1 m

Discuss qualitatively the response characteristics of the system if u is a unit step input when using Design A and Design B. What is the primary difference in the response characteristics between these two designs?

c) Perform the same analysis as in b), except now let u be a unit impulse. What is the primary difference in the response characteristics between these two designs?

Problem 2

Consider a unity-feedback system whose open-loop (feedforward) part is a proportional controller (P controller) followed by a third-order system

$$G(s) = \frac{s^2 + s + 1}{s^3 + s^2 + k_1 s + k_2}.$$

Here k_1 and k_2 are constant system parameters. Suppose k_1 and k_2 are unknown with

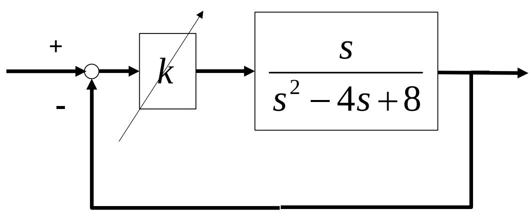
$$1 \leq k_1 \leq 2, \; 3 \leq k_2 \leq 4.$$

(a) Determine the range of the choice of the proportional gain K which guarantees the stability of the closed-loop system.

(b) Choose K such that the magnitude of the steady-state error e_∞ of the system, where the reference input is a unit-step function, is guaranteed to be less than 0.01.

Problem 3

Consider the unity feedback of a dynamical system $G(s) = \dfrac{s}{s^2 - 4s + 8}$ with a loop gain k (>0) as shown below.



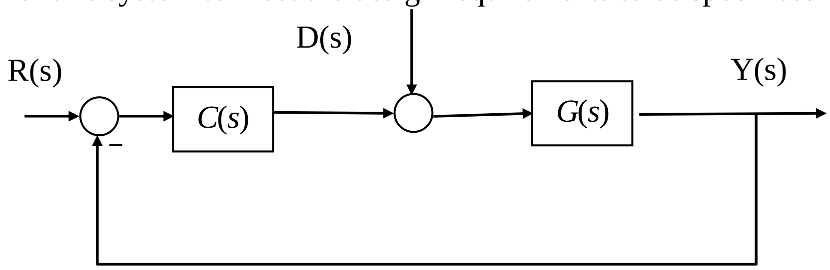
a) Sketch the root-locus plot of the system. Determine the angles of departure from complex poles, intersection of asymptotes, break-in/away points, if they exist.

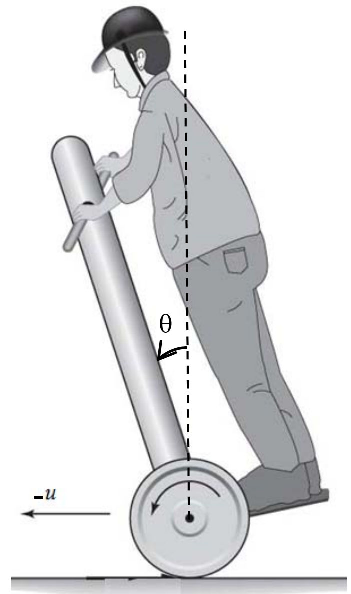
b) (b-1) Determine k such that the closed-loop system is *undamped*, and (b-2) find the frequency of the undamped response of the closed-loop system.

c) (c-1) Determine k such that the closed-loop system is *critically-damped*, and (c-2) sketch the unit-step response of the closed-loop system. (NOTE: a general sketch is acceptable and detailed mathematical derivations are not required. However, you should provide sufficient discussion to characterize the step response.)

Problem 4

The Transfer function of the Segway transportation device shown below is given by $\Theta(s)/U(s)=G(s)=s/(s^2-a^2)$ for some $a>0$ where u is the linear velocity input and θ is the angular displacement output You are asked to design a feedback controller C(s) (see the block diagram below) for this system to meet the design requirements to be specified.





a) Is the open-loop system $G(s)$ stable or not? Why?
b) Design a controller of your choice (e.g., P,PD,PI,PID,Lead,Lag,…) such that the closed loop system has a settling time of at most $4/a$ seconds, a Phase Margin (PM) of at least 60 degrees, and no steady state error for $r=0$.
c) Find the steady state amplitude of $y(t)$ in response to the sinusoidal input disturbance $d(t)=d\cos\omega t$ for any $\omega,d\geq 0$ and $r=0$ using the controller you found in (b).